

# Appendix

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## A MATHPILE Datasheet

<b>MOTIVATION</b>	
<b>For what purpose was the dataset created?</b>	Developed in a context where datasets like Google’s Minerva and OpenAI’s MathMix are not open-sourced, MATHPILE aims to counter this trend by enriching the open-source community and enhancing mathematical language modeling with its (relatively) large-scale, math-centric, diverse, high-quality dataset. It can be used on its own or cooperated with general domain corpora like books, and Github code, to improve the reasoning abilities of language models.
<b>Who created the dataset and on behalf of which entity?</b>	MATHPILE was created by the authors of this work.
<b>Who funded the creation of the dataset?</b>	The creation of MATHPILE was funded by GAIR Lab, SJTU.
<b>Any other comment?</b>	None.
<b>COMPOSITION</b>	
<b>What do the instances that comprise the dataset represent?</b>	MATHPILE is comprised of text-only documents, encompassing a broad range of sources. These include academic papers from arXiv, educational materials such as textbooks and lecture notes, definitions, theorems and their proofs, informative articles from Wikipedia, interactive Q&A content from StackExchange community users, and webpages sourced from Common Crawl. All these instances are math-focused.
<b>How many instances are there in total?</b>	MATHPILE contains about 903 thousand of documents, or around 9.5 billion tokens.
<b>Does the dataset contain all possible instances or is it a sample (not necessarily random) of instances from a larger set?</b>	MATHPILE is curated from a diverse array of sources, including arXiv, Textbooks, Wikipedia, StackExchange, ProofWiki, and Common Crawl. However, it doesn’t encompass all instances from these sources. We have implemented a rigorous data processing pipeline, which involves steps like preprocessing, prefiltering, language identification, cleaning, filtering, and deduplication. This meticulous approach is taken to guarantee the high quality of the content within MATHPILE .
<b>What data does each instance consist of?</b>	Each instance in MATHPILE is a text-only document, uniquely identified by its source, labeled under Subset. These instances are enriched with metadata, such as the score from language identification, the ratio of symbols to words, and their respective file paths. Note that instances from the StackExchange are composed of a question and its accompanying answers, each with their own set of meta data, including community users. To illustrate them, we provide specific examples for each source, ranging from Figure 4 to Figure 10.
<b>Is there a label or target associated with each instance?</b>	No.
<b>Is any information missing from individual instances?</b>	No.

<b>Are relationships between individual instances made explicit?</b>	No.
<b>Are there recommended data splits?</b>	No.
<b>Are there any errors, sources of noise, or redundancies in the dataset?</b>	Despite our rigorous efforts in cleaning, filtering out low-quality content, and deduplicating documents, it's important to acknowledge that a small fraction of documents in MATHPILE might still fall short of our quality standards, particularly those sourced from web pages.
<b>Is the dataset self-contained, or does it link to or otherwise rely on external resources?</b>	Yes, MATHPILE is self-contained.
<b>Does the dataset contain data that might be considered confidential?</b>	No.
<b>Does the dataset contain data that, if viewed directly, might be offensive, insulting, threatening, or might otherwise cause anxiety?</b>	We do not expect offensive content despite our significant efforts in cleaning and filtering. But, we can not fully guarantee this.

#### COLLECTION

<b>How was the data associated with each instance acquired?</b>	Our data is primarily sourced from the arXiv website and the Internet Archive. The CommonCrawl data originates from SlimPajama. The textbooks included are manually collected, with quality checks performed on publicly available textbooks from various internet sources.
<b>What mechanisms or procedures were used to collect the data?</b>	Refer to § 2 for details on how they collect data.
<b>If the dataset is a sample from a larger set, what was the sampling strategy?</b>	We strive to use the most recent data dumps available and then selectively choose high-quality documents that are closely related to mathematics.
<b>Who was involved in the data collection process and how were they compensated?</b>	Authors from this paper were involved in collecting it and processing it.
<b>Over what timeframe was the data collected?</b>	MATHPILE encompasses documents created between 2007 and August 2023. Note that some documents and textbooks included may be created in the previous century.
<b>Were any ethical review processes conducted?</b>	No.

#### PREPROCESSING

<b>Was any preprocessing/-cleaning/labeling of the data done?</b>	Yes, during our data collection phase, we conducted extensive filtering and cleansing procedures, detailed in § 2. After the completion of data collection, we conducted further steps including language identification, additional cleaning and filtering, deduplication, and leakage detection in benchmark datasets. Subsequently, we removed any contaminated examples identified through this process. See § 3 for details.
<b>Was the “raw” data saved in addition to the preprocessed/-cleaned/labeled data?</b>	Yes.
<b>Is the software that was used to preprocess/clean/label the data available?</b>	Yes, scripts are open-sourced at <a href="https://github.com/GAIR-NLP/MathPile/tree/main/src">https://github.com/GAIR-NLP/MathPile/tree/main/src</a>

#### USES

<b>Has the dataset been used for any tasks already?</b>	Yes, this data has been used to develop mathematical language models.
<b>Is there a repository that links to any or all papers or systems that use the dataset?</b>	No. This dataset is currently utilized in the following research papers: (1) JiuZhang 3.0: Efficiently Improving Mathematical Reasoning by Training Small Data Synthesis Models. (2) Task Oriented In-Domain Data Augmentation. (3) Great Memory, Shallow Reasoning: Limits of $k$ NN-LMs. (4) BAM! Just Like That: Simple and Efficient Parameter Upcycling for Mixture of Experts. (5) SciDFM: A Large Language Model with Mixture-of-Experts for Science. (6) MIND: Math Informed syNthetic Dialogues for Pretraining LLMs and so on.
<b>What (other) tasks could the dataset be used for?</b>	MATHPILE was developed to enhance language modeling, offering significant benefits for a variety of mathematical reasoning tasks.
<b>Is there anything about the composition of the dataset or the way it was collected and preprocessed/-cleaned/labeled that might impact future uses?</b>	Our cleaning and filtering processes, while thorough, may not be entirely optimal, potentially leading to the exclusion of some valuable documents. Additionally, MATHPILE is specifically tailored for English, which limits its applicability in multilingual contexts.
<b>Are there tasks for which the dataset should not be used?</b>	Any tasks which may considered irresponsible or harmful.

#### DISTRIBUTION

<b>Will the dataset be distributed to third parties outside of the entity on behalf of which the dataset was created?</b>	Yes, MATHPILE has been made available through the HuggingFace Hub ( <a href="https://huggingface.co/datasets/GAIR/MathPile">https://huggingface.co/datasets/GAIR/MathPile</a> ).
<b>How will the dataset will be distributed?</b>	MATHPILE has been made available through the HuggingFace Hub ( <a href="https://huggingface.co/datasets/GAIR/MathPile">https://huggingface.co/datasets/GAIR/MathPile</a> ).
<b>When will the dataset be distributed?</b>	The MATHPILE will be available after this paper is made public.

<b>Will the dataset be distributed under a copyright or other intellectual property (IP) license, and/or under applicable terms of use (ToU)?</b>	If the source data of MATHPILE is governed by a license more restrictive than CC BY-NC-SA 4.0, MATHPILE adheres to that stricter licensing. In all other cases, it operates under the CC BY-NC-SA 4.0 license. If any data owner objects to the use of their data, we are willing to take appropriate action immediately, including removing the relevant data.
<b>Have any third parties imposed IP-based or other restrictions on the data associated with the instances?</b>	Not to our knowledge.
<b>Do any export controls or other regulatory restrictions apply to the dataset or to individual instances?</b>	Not to our knowledge.
<b>MAINTENANCE</b>	
<b>Who will be supporting/hosting/maintaining the dataset?</b>	MATHPILE will be hosted on the HuggingFace Hub.
<b>How can the owner/curator/manager of the dataset be contacted?</b>	stefanpengfei@gmail.com    zzwang.nlp@gmail.com
<b>Is there an erratum?</b>	No.
<b>Will the dataset be updated?</b>	Yes, it is currently a work in progress and updates are ongoing.
<b>If others want to extend/augment/build on/contribute to the dataset, is there a mechanism for them to do so?</b>	No.

Table 6: Datasheet for MATHPILE , following Gebru et al. [21].

## B Ethics Statement

In the collection and creation of MATHPILE , we strictly adhered to all copyright and licensing requirements of the data sources. Specifically, we gathered a large amount of data from the internet, including mathematical textbooks, web pages, and community Q&A content, ensuring that the use of these data complies with the original licensing terms. Wikipedia, ProofWiki, and StackExchange are licensed under CC BY-SA (2.5, 3.0 or 4.0). Textbooks and arXiv are licensed under CC BY 4.0, CC BY-SA 4.0, CC BY-NC-SA 4.0 and others. Common Crawl follows the Common Crawl Foundation Terms of Use and C4 license. The final open-source MATHPILE dataset is released under the CC BY-NC-SA 4.0 license. If the source data’s license is more restrictive than CC BY-NC-SA 4.0, we adopt the stricter license.

However, during the collection of some data, such as publicly available and open-source textbooks, we did not obtain explicit consent from each author. We recognize that this may involve potential copyright issues. Therefore, we have implemented the following measures to mitigate and manage these risks:

1. **Strict Selection of Data Sources:** We prioritize selecting data sources that are clearly marked with open licenses or public domain status, avoiding the use of content explicitly marked as copyright-protected or prohibited from distribution.
2. **Adherence to Fair Use Principles:** When using copyrighted and non-commercially licensed content, we adhere to the principles of fair use, aiming to promote scientific research and educational purposes rather than commercial purposes, thereby not affecting the market value of the original content.
3. **Acceptance of Feedback from Users and Content Authors:** We welcome feedback from data users and authors at any time to request the removal or modification of their data.

MATHPILE has been carefully curated and processed to minimize any potential ethical concerns. We also explicitly state that if any data owner objects to the use of their data, we are willing to take appropriate action immediately, including removing the relevant data. Through these measures, we strive to ensure the diversity and richness of the collected data while complying with relevant copyright and licensing regulations, thereby reducing potential legal risks. We bear full responsibility for any potential violations of rights or licensing issues that may arise from this dataset.

## C Examples of MATHPILE

We provide some illustrative examples from each source in MATHPILE , as shown in Figure 4 to Figure 10.

### A document from MATHPILE-CommonCrawl

**Text:**

Are there optimizers where it is possible to specify ordinal ranking of parameters?

Assume that  $f$  is smooth ( $n$ -th order differentiable in each of the parameters).

An approach I often use when applying unconstrained optimisation algorithms to constrained problems is to transform the parameter space such that the constraints cannot be violated.

Of course this results in  $\theta_1^* \geq \theta_2^* \geq \theta_3^*$  which isn't quite what you asked for. To get a strict ranking you'll need to bump  $x_1 - x_2^2$  and  $x_1 - x_2^2 - x_3^2$  down at the last digit of precision.

this spake a.k.this spake a.k.

These variants of your constraints are linear, so provided that your function  $f$  is well-behaved (smooth, easy to calculate, easy to compute derivatives, derivatives are well-conditioned, etc.), any constrained optimization solver should be able to solve your problem without issue.

Not the answer you're looking for? Browse other questions tagged optimization constrained-optimization or ask your own question.

Does the amount of correlation of model parameters matter for nonlinear optimizers?

Optimization of a blackbox function with an equality constraint?

...

**Subset:** CommonCrawl

**meta:**

language\_detection\_score: 0.8670,  
char\_num\_after\_normalized: 926,  
contain\_at\_least\_two\_stop\_words: True,  
ellipsis\_line\_ratio: 0.0,  
idx: 383668,  
lines\_start\_with\_bullet\_point\_ratio: 0.0,  
mean\_length\_of\_alpha\_words: 5.0870,  
non\_alphabetical\_char\_ratio: 0.0,  
symbols\_to\_words\_ratio: 0.0,  
uppercase\_word\_ratio: 0.0060,  
...

Figure 4: An example Common Crawl document in MATHPILE

## A document from MATHPILE -Wikipedia

### Text:

#### # Inner Automorphism

In abstract algebra, an **inner automorphism** is an automorphism of a group, ring, or algebra given by the conjugation action of a fixed element, called the *conjugating element*. They can be realized via simple operations from within the group itself, hence the adjective "inner". These inner automorphisms form a subgroup of the automorphism group, and the quotient of the automorphism group by this subgroup is defined as the outer automorphism group.

#### ## Definition

If  $G$  is a group and  $g$  is an element of  $G$  (alternatively, if  $G$  is a ring, and  $g$  is a unit), then the function

$$\begin{aligned}\varphi_g : G &\rightarrow G \\ \varphi_g(x) &:= g^{-1}xg\end{aligned}$$

is called **(right) conjugation by  $g$**  (see also conjugacy class). This function is an endomorphism of  $G$ : for all  $x_1, x_2 \in G$ ,

$$\varphi_g(x_1 x_2) = g^{-1}x_1 x_2 g = (g^{-1}x_1 g)(g^{-1}x_2 g) = \varphi_g(x_1)\varphi_g(x_2),$$

where the second equality is given by the insertion of the identity between  $x_1$  and  $x_2$ . Furthermore, it has a left and right inverse, namely  $\varphi_{g^{-1}}$ . Thus,  $\varphi_g$  is bijective, and so an isomorphism of  $G$  with itself, i.e., an automorphism. An **inner automorphism** is any automorphism that arises from conjugation.[1]

When discussing right conjugation, the expression  $g^{-1}xg$  is often denoted exponentially by  $x^g$ . This notation is used because composition of conjugations satisfies the identity:  $(x^{g_1})^{g_2} = x^{g_1 g_2}$  for all  $g_1, g_2 \in G$ . This shows that right conjugation gives a right action of  $G$  on itself.

#### ### Inner and Outer Automorphism Groups

The composition of two inner automorphisms is again an inner automorphism, and with this operation, the collection of all inner automorphisms of  $G$  is a group, the inner automorphism group of  $G$  denoted  $\text{Inn}(G)$ .

$\text{Inn}(G)$  is a normal subgroup of the full automorphism group  $\text{Aut}(G)$  of  $G$ . The outer automorphism group,  $\text{Out}(G)$ , is the quotient group

$$\text{Out}(G) = \frac{\text{Aut}(G)}{\text{Inn}(G)}.$$

The outer automorphism group measures, in a sense, how many automorphisms of  $G$  are not inner. Every non-inner automorphism yields a non-trivial element of  $\text{Out}(G)$ , but different non-inner automorphisms may yield the same element of  $\text{Out}(G)$ .

Saying that conjugation of  $x$  by  $a$  leaves  $x$  unchanged is equivalent to saying that  $a$  and  $x$  commute:

$$a^{-1}xa = x \iff xa = ax.$$

Therefore, the existence and number of inner automorphisms that are not the identity mapping is a kind of measure of the failure of the commutative law in the group (or ring).

An automorphism of a group  $G$  is inner if and only if it extends to every group containing  $G$ . [2]

...

### Subset: Wikipedia

#### meta:

```
language_detection_score: 0.7236,
char_num_after_normalized: 5794,
contain_at_least_two_stop_words: True,
ellipsis_line_ratio: 0.0,
lines_start_with_bullet_point_ratio: 0.0,
mean_length_of_alpha_words: 4.2245,
mimetype: text/html,
page_index: 48171,
page_path: A/Inner_automorphism,
page_title: Inner automorphism,
non_alphabetical_char_ratio: 0.1422,
symbols_to_words_ratio: 0.0,
uppercase_word_ratio: 0.0871,
...
```

Figure 5: An example Wikipedia document in MATHPILE



A document from MATHPILE-Textbooks

Text:

# LINEAR TORIC FIBRATIONS

SANDRA DI ROCCO

## INTRODUCTION TO TORIC FIBRATIONS

Definition 1.1. A toric fibration is a surjective flat map  $f : X \rightarrow Y$  with connected fibres where

- (a)  $X$  is a toric variety
- (b)  $Y$  is a normal algebraic variety
- (c)  $\dim(Y) < \dim(X)$ .

Remark 1.2. Observe that if  $f : X \rightarrow Y$  is a toric fibration then  $Y$  and a general fiber  $F$  are also toric varieties. Moreover if  $X$  is smooth, respectively  $\mathbb{Q}$ -factorial then so is  $Y$  and  $F$ .

Combinatorial characterization. A toric fibration has the following combinatorial characterization (see [EW, Chapter VI] for further details). Let  $X = X_\Sigma$ , where  $\Sigma \subset N \cong \mathbb{Z}^n$ , be a toric variety of dimension  $n$  and let  $i : \Delta \hookrightarrow N$  a sublattice.

Proposition 1.3. [EW] The inclusion  $i$  induces a toric fibration if and only if:

- (a)  $\Delta$  is a primitive lattice, i.e.  $(\Delta \otimes \mathbb{R}) \cap N = \Delta$ .
- (b) For every  $\sigma \in \Sigma(n)$ ,  $\sigma = \tau + \eta$ , where  $\tau \in \Delta$  and  $\eta \cap \Delta = \{0\}$  (i.e.  $\Sigma$  is a splitfan).

We briefly outline the construction. The projection  $\pi : N \rightarrow N/\Delta$  induces a map of fans  $\Sigma \rightarrow \pi(\Sigma)$  and thus a map of toric varieties  $f : X \rightarrow Y$ . The general fiber  $F$  is a toric variety defined by the fan  $\Sigma_F = \{\sigma \in \Sigma \cap \Delta\}$ .

When the toric variety  $X$  in a toric fibration is polarized by an ample line bundle  $L$  we will call the pair  $(f : X \rightarrow Y, L)$  a polarized toric fibration. Observe that the polarized toric varieties  $(X, L)$  and  $(F, L|_F)$ , for a general fiber  $F$ , define lattice polytopes  $P_{(X,L)}, P_{(F, L|_F)}$ . The polytope  $P_{(X,L)}$  is in fact a "twisted sum" of a finite number of lattice polytopes fibering over  $P_{(F, L|_F)}$ . Definition 1.4. Let  $R_0, \dots, R_k \subset \Delta$  be polytopes. Let  $\pi : M \rightarrow \Lambda$  be a surjective map of lattices such that  $\pi(R_i) = v_i$  and the  $v_0, \dots, v_k$  are distinct vertices of  $\text{Conv}(v_0, \dots, v_k)$ . We will call a Cayley  $\pi$ -twisted sum (or simply a Cayley sum) of  $R_0, \dots, R_k$  a polytope which is affinely isomorphic to  $\text{Conv}(R_0, \dots, R_k)$ . We will denote it by:

$$[R_0 \star \dots \star R_k]_\pi$$

If the polytopes  $R_i$  are additionally normally equivalent, i.e. they define the same normal fan  $\Sigma_Y$ , we will denote the Cayley sum by:

$$\text{Cayley}(R_0, \dots, R_k)_{(\pi, Y)}.$$

These are the polytopes that are associated to a polarized toric fibration. Consider a sublattice  $i : \Delta \hookrightarrow N$  and the dual lattice surjection  $\pi : M \rightarrow \Lambda$ .

Proposition 1.5. [CDR08] The sublattice  $i : \Delta \hookrightarrow N$  induces a polarized toric fibration  $(f : X \rightarrow Y, L)$  if and only if  $P_{(X,L)} = \text{Cayley}(R_0, \dots, R_k)_{(\pi, Y)}$  for some normally equivalent polytopes  $R_0, \dots, R_k$ .

The polarized general fiber  $(F, L|_F)$  corresponds to the polarized toric variety associated to the polytope  $P_{(F, L|_F)} = \text{Conv}(v_0, \dots, v_k)$  and the polytopes  $R_0, \dots, R_k$  define the embeddings of the invariant sections polarized by the restrictions of  $L$ .

Example 1.6. Consider the Hirzebruch surface  $\mathbb{F}_1 = \text{Bl}_p(\mathbb{P}^2) = \mathbb{P}(\mathcal{O}_{\mathbb{P}^1} \oplus \mathcal{O}_{\mathbb{P}^1}(1))$  polarized by the tautological line bundle  $\xi = 2\phi^*(\mathcal{O}_{\mathbb{P}^2}(1)) - E$  where  $\phi$  is the blow-up map and  $E$  the exceptional divisor. The associated polytope is  $P = \text{Cayley}(\Delta_1, 2\Delta_1)$ .

FIGURE 1. The Hirzebruch surface  $\mathbb{P}(\mathcal{O}_{\mathbb{P}^1} \oplus \mathcal{O}_{\mathbb{P}^1}(1))$

Example 1.7. More generally:

- when  $\pi(P) = \Delta_i$  the polytope  $\text{Cayley}(R_0, \dots, R_k)_{(\pi, Y)}$  defines the variety  $\mathbb{P}(L_0 \oplus \dots \oplus L_k)$ , where the  $L_i$  are ample line bundles on the toric variety  $Y$ , polarized by the tautological bundle  $\xi$ . In particular  $L|_F = \mathcal{O}_{\mathbb{P}^t}(1)$ .

- When  $\pi(P)$  is a simplex (not necessarily smooth)  $\text{Cayley}(R_0, \dots, R_k)_{(\pi, Y)}$  defines a Mori-type fibration. A fibration whose general fiber has Picard rank one. - When  $\pi(P) = s\Delta_i$  then again the variety has the structure of a  $\mathbb{P}^t$ -fibration whose general fiber  $F$  is embedded via an  $s$ -Veronese embedding:  $(F, L|_F) = (\mathbb{P}^t, \mathcal{O}_{\mathbb{P}^t}(s))$ .

For general Cayley sums,  $[R_0 \star \dots \star R_k]_\pi$ , one has the following geometrical interpretation. Let  $(X, L)$  be the associated polarized toric variety and let  $Y$  be the toric variety defined by the Minkowski sum  $R_0 + \dots + R_k$ . The fan defining  $Y$  is a refinement of the normal fan of  $R_i$  for  $i = 0, \dots, k$ . Consider the associated birational maps  $\phi_i : Y \rightarrow Y_i$ , where  $(Y_i, L_i)$  is the polarized toric variety defined by the polytope  $R_i$ . The line bundles  $H_i = \phi_i^*(L_i)$  are nef line bundles on  $Y$ . Denote by the same symbol the maps of fans  $\phi_i : \Sigma_Y \rightarrow \Sigma_{Y_i}$ . Define then the fan:

$$\Sigma_Z : \left\{ \phi_i^{-1}(\sigma_j) \times \eta_l, \text{ for all } \sigma_j \in \Sigma_{Y_i}, \eta_l \in \Sigma_\Delta \right\}$$

where  $\Lambda = \text{Conv}(v_0, \dots, v_k)$ . It is a refinement of  $\Sigma_X$  and thus the defining variety  $Z$  is birational to  $X$ . Moreover it is a split fan and thus it defines a toric fibration  $f : Z \rightarrow Y$ . The Cayley sum  $[R_0 \star \dots \star R_k]_\pi$  is the polytope defined by the nef line bundle  $\phi^*(L)$ , and the polytopes  $R_i$  are the polytopes defined by the nef line bundles  $H_i$  on the invariant sections.

Historical Remark. The definition of a Cayley polytope originated by what is "classically" referred to as the Cayley trick. We first recall the definition of Resultant and Discriminant. Let  $f_1(x), \dots, f_n(x)$  be a system of  $n$  polynomials in  $n$  variables  $x = (x_1, \dots, x_n)$  supported on  $A \subset \mathbb{Z}^n$ . This means that  $f_i = \Pi_{\alpha_j \in A} c_j x^{\alpha_j}$ . The resultant (of  $A$ ),  $R_A(c_j)$ , is a polynomial in the coefficients  $c_j$ , which vanishes whenever the corresponding polynomials have a common zero.

The discriminant of a finite subset  $A$ ,  $\Delta_{\mathcal{A}}$ , is also a polynomial  $\Delta_{\mathcal{A}}(c_j)$  in the variables  $c_j \in A$  which vanishes whenever the corresponding polynomial has a multiple root.

Theorem 1.8. [GKZ][Cayley Trick] The A-resultant of the system  $f_1, \dots, f_n$  equals the Adiscriminant of the polynomial:

$$p(x, y) = f_i(x) + \sum_2^n y_{i-1} f_i(x).$$

Let  $R_i = N(f_i) \subset \mathbb{R}^n$  be the Newton polytopes of the polynomials  $f_i$ . The Newton polytope of the polynomial  $p(x, y)$  is the Cayley sum  $[R_1 \star \dots \star R_n]_\pi$ , where  $\pi : \mathbb{R}^{2n-1} \rightarrow \mathbb{R}^{n-1}$  is the natural projection such that  $\pi([R_1 \star \dots \star R_n]_\pi) = \Delta_{n-1}$ .

...

Subset: Textbooks

meta:

book\_name: Linear Toric Fibrations\_Sandra Di Rocco,

type: Notes,

...

```

A document from MATHPILE-ProofWiki

Text:
\section[Test for Submonoid]
Tags: Abstract Algebra, Monoids

\begin{theorem}
To show that \struct {T, circ} is a submonoid of a monoid \struct {S, circ}, we need to show that:
:(1):  $T \subseteq S$ 
:(2): \struct {T, circ} is a magma (that is, that it is closed)
:(3): \struct {T, circ} has an identity.
\end{theorem}

\begin{proof}
From Subsemigroup Closure Test, (1) and (2) are sufficient to show that \struct {T, circ} is a subsemigroup of \struct {S, circ}.
Demonstrating the presence of an identity is then sufficient to show that it is a monoid. {\qed}
Category:Monoids
\end{proof}

...

Subset: ProofWiki

meta:
type: Theorem_Proof,
...

```

Figure 7: An example ProofWiki (a theorem and its proof) document in MATHPILE

```

A document from MATHPILE-ProofWiki

Text:
\begin{definition}[Definition:That which produces Medial Whole with Medial Area/Whole]
Let  $a, b \in \mathcal{R}_{>0}$  be (strictly) positive real numbers such that  $a > b$ .
Let  $a - b$  be a straight line which produces with a medial area a medial whole.
The real number  $a$  is called the "whole" of the straight line which produces with a medial area a medial whole.
Category:Definitions/Euclidean Number Theory
\end{definition}

Subset: ProofWiki

meta:
type: Definition,
...

```

Figure 8: An example ProofWiki (definition) document in MATHPILE

## A document from MATHPILE-arXiv

**Text:**

```
\begin{document}
\title{Coherence freeze in an optical lattice investigated via pump-probe spectroscopy}
\author{Samansa Maneshi}
\email[|]{smaneshi@physics.utoronto.ca}
\author{Chao Zhuang}
\author{Christopher R. Paul}
\author{Luciano S. Cruz}
\altaffiliation[Current address: ]{UFABC, São Paulo, Brazil.}
\author{Aephraim M. Steinberg}
\affiliation{Centre for Quantum Information & Quantum Control and Institute for Optical Sciences,
Department of Physics, University of Toronto, Canada }
\date{\today}
\pacs{37.10.Jk, 03.65.Yz, 03.67.-a, 42.50.Md}

\begin{abstract}
Motivated by our observation of fast echo decay and a surprising coherence freeze, we have developed a pump-probe spectroscopy technique for vibrational states of ultracold  $^{85}\text{Rb}$  atoms in an optical lattice to gain information about the memory dynamics of the system. We use pump-probe spectroscopy to monitor the time-dependent changes of frequencies experienced by atoms and to characterize the probability distribution of these frequency trajectories. We show that the inferred distribution, unlike a naive microscopic model of the lattice, correctly predicts the main features of the observed echo decay.
\end{abstract}

\maketitle

Characterizing decoherence mechanisms is a crucial task for experiments aiming to control quantum systems, e.g., for quantum information processing (QIP). In this work, we demonstrate how two-dimensional (2D) pump-probe spectroscopy may be extended to provide important information on these mechanisms. As a model system, we study quantum vibrational states of ultracold atoms in an optical lattice. In addition to being a leading candidate system for QIP \cite{BrennenJaksch}, optical lattices are proving a versatile testing ground for the development of quantum measurement and control techniques \cite{OMandel, Anderlini} and a powerful tool for quantum simulations, e.g. the study of Anderson localization and the Hubbard model \cite{MottAnderson}.

In our experiment, we study the vibrational coherence of  $^{85}\text{Rb}$  atoms trapped in a shallow one-dimensional standing wave. Through our 2D pump-probe technique, we obtain detailed microscopic information on the frequency drift experienced by atoms in the lattice, enabling us to predict the evolution of coherence. Since the pioneering development of the technique in NMR \cite{JeenerErnst}, 2D spectroscopy has been widely used to obtain high-resolution spectra and gain information about relaxations, couplings, and many-body interactions, in realms ranging from NMR \cite{Ernst} to molecular spectroscopy \cite{MukamelJonas, Hybl, Brixner, MillerNature} to semiconductor quantum wells \cite{Cundiff, KWStone}. Here, we show that similar powerful techniques can be applied to the quantized center-of-mass motion of trapped atoms, and more generally, offer a new tool for the characterization of systems in QIP and quantum control.

\begin{figure}
\caption{(Color online) Two typical measurements of echo amplitude vs. time. The echo pulse and the observed echo envelope are centered at times  $t_p$  and  $2t_p$ , respectively. After an initial decay, echo amplitude stays constant for about  $1\text{ms}$  forming a plateau, before decaying to zero. The average lattice depths are  $20E_R$  (circles) and  $18E_R$  (squares.)
\label{fig1}
\end{figure}

We have previously measured the evolution of coherence between the lowest two vibrational states of potential wells \cite{Ours}. The dephasing time is about  $0.3\text{ms}$  ( $T_2^*$ ). This dephasing is partly due to an inhomogeneous distribution of lattice depths as a result of the transverse Gaussian profile of the laser beams. To measure the homogeneous decoherence time ( $T_2$ ), we perform pulse echoes, measuring the echo amplitude as a function of time \cite{Ours}. Figure \ref{fig1} shows two typical measurements of echo amplitude carried out on different dates under slightly different conditions such as different average lattice depths and different dephasing times. The echo amplitude initially decays with a time constant of about  $0.7\text{ms}$ , which is much faster than the photon scattering time ( $\sim 60\text{ms}$ ) in the lattice. It then exhibits a  $1\text{ms}$ -long coherence freeze followed by a final decay. Absent real decoherence on the short time scale of  $1\text{ms}$ , only loss of frequency memory would inhibit the appearance of echoes. This loss comes about when atoms experience time-varying frequencies. We use 2D pump-probe spectroscopy to monitor this frequency drift. Our 2D pump-probe spectroscopy is essentially a version of spectral hole-burning for vibrational states. By monitoring the changes in the hole spectrum as a function of time we gain information on the atoms' frequency drift. Information obtained from our 2D spectra enables us to characterize the temporal decay of frequency memory and through our simulations we find that "coherence freeze" is related to the shape of this memory loss function.

Similar plateaus in echo decay and a two-stage decay of echo amplitude have been observed in a Cooper-pair box \cite{Nakamura}, for a single electron spin in a quantum dot \cite{Vandersypen} and for electron spins in a semiconductor \cite{SClark}. Those plateaus or two-stage decays have been either explained through \textit{a priori} models or simply described phenomenologically. Here, we are introducing an experimental technique to directly probe the origin of plateaus.

The periodic potential in our experiment is formed by interfering two laser beams blue-detuned by  $25\text{GHz}$  from the D2 transition line,  $F = 3 \rightarrow F' = 4$  ( $\lambda = 780\text{nm}$ ), thus trapping atoms in the regions of low intensity, which minimizes the photon scattering rate and the transverse forces. The two laser beams intersect with parallel linear polarizations at an angle of  $\theta = (49.0 \pm 0.2)^\circ$ , resulting in a spacing of  $L = (0.930 \pm 0.004)\mu\text{m}$  between the wells. Due to gravity, the full effective potential also possesses a "tilt" of  $2.86E_R$  per lattice site, where  $E_R = \frac{\hbar^2}{8mL^2}$  is the effective lattice recoil energy. The photon scattering time in our experiment is  $\approx 60\text{ms}$  and the Landau-Zenner tunneling times for transitions from the lowest two levels are greater than  $160\text{ms}$ . Atoms are loaded to the lattice during a molasses cooling stage and prepared in the ground vibrational state by adiabatic filtering \cite{StefanQPT}. Due to the short coherence length of atoms in optical molasses ( $60\text{nm}$  at  $10\mu\text{K}$ ), there is no coherence between the wells. We measure populations of atoms in the ground vibrational, the first excited, and the (lossy) higher excited states  $P_1$ ,  $P_2$ , and  $P_L$ , respectively, by fluorescence imaging of the atomic cloud after adiabatic filtering \cite{StefanQPT}.

...

Subset: arXiv



---


meta:
id: 1005.2635,
language_detection_score: 0.8389,
...
```

Figure 9: An example arXiv document in MATHPILE

## A document from MATHPILE-StackExchange

### Question:

Title: Are fractions hard because they are like algebra?

Body:

It occurs to me that to really understand the ways that people work with fractions on paper requires a good grasp of the ideas that numbers have multiple representations and that expressions can be manipulated in various ways without changing the number they represent. These are essentially algebraic ideas.

For example, adding fractions requires us to rewrite the fractions in a different form, and then essentially factorise the expression. This is the same as rearranging expressions in algebra. Dividing fractions requires us to rerepresent an operation like  $\div \frac{2}{3}$  as  $\times \frac{3}{2}$ . This is the same as realising the connection between operations that you use to solve equations in algebra. And cancelling down before multiplying is very sophisticated rewriting relying on various associative and commutative laws.

So it seems that we are really asking children to think in algebraic ways in order to understand fraction calculations well. This would seem to me to be a good reason why children and adults find it hard - they need more scaffolding in some abstract ideas.

Is this a reasonable theory and has anyone written about this algebra-fractions connection before? To be clear, I am not asking if this is the only reason fractions are hard, but if there is any discussion out there to draw parallels between learning algebra and learning to manipulate fractions.

Id: 7826

Score: 17

Tags: <algebra><fractions>

LastEditorDisplayName: None

OwnerDisplayName: None

ClosedDate: None

FavoriteCount: None

language\_detection\_score: 0.9558

...

### Answers:

Body: Not sure about paper references. One reason why people don't understand fractions is because they are seemingly illogical.

You score one basket out of three 1/3.

A little while later you try again and score 1/2. Clearly you have scored 2/5 shots? In many ways this is the correct answer. So why shouldn't  $\frac{1}{3} + \frac{1}{2} = \frac{2}{5}$

People generally don't understand equivalent fractions. It is strange for one farmer to say there are 4 sheep and another to say there are 8/2 sheep in the same field. People assume that the number 4 does what it says on the tin and is how we always describe 4 ness of something. They don't understand equivalence.

Partly to blame is treating fractions like conjuring tricks. If this is the question...do this, if this is the question ...do another uncorrelated thing. I asked my class (who seemingly could compute  $\frac{2}{3} \times \frac{3}{5}$  correctly) to draw me a picture

instead of just multiplying. No one could do it yet they all said "but it's *frac*15 you times the top and the bottom!"

I think drawing fractions is extremely useful. Draw  $\frac{2}{3} \div 2$  or  $2 \div \frac{2}{3}$ . It's not easy but I find students develop robustness eventually and begin to abstract themselves.

Id: 7827,

Score: 9,

is\_accepted\_answer: False,

language\_detection\_score: 0.9599,

Body: The obvious (to me) source of difficulty is that fractions are just plain complicated, more so than almost anything else in elementary education. You have to operate with a pair of numbers, instead of a single one, and you have to keep the order straight. Adding is quite complicated in its own right. Things are further complicated by rules about least common denominators and least terms.

It's a little unclear about the question's emphasis on algebra. Any sort of general rule or operation in arithmetic must have a connection to algebra, but I do not see what is intrinsically difficult about algebra that relates to numeric fractions. Certainly some parts of algebra are hard, and some parts harder than others, algebraic fractions among them. It seems to me that fractions are difficult because it's easy to confuse the various bits. Even when you've got them straight, they're noticeably slower to use, take concentration, and when things have such cognitive demands, they're harder to think with.

Conceptually, they're a little bit odd, which is probably distracting until you get used to them. What they represent do not seem to apply to the same things that (whole) numbers do. Evidently fractions are not considered in this passage:

In that city, which was the oldest in the world, the cat was an object of veneration. Its worship was the religion of the country. The multiplication and addition of cats were a perpetual instruction in arithmetic. Naturally, any inattention to the wants of a cat was punished with great severity in this world and the next... - A. Bierce, "A Revolt of the Gods"

Now to have one-and-a-half cats seems a very different thing than to have three halves. In the former case, there's a good chance that the one cat you have will be alive and purring, while the same could not possibly be said about any of the halves. No doubt such lessons are considered blasphemous in that city. While many things may be divided into parts - cars are a better example than cats - not many can be divided into equivalent parts that can be used as a basis for fractions. As we get used to fractions, as well as real numbers, we are taught to ignore this and accept statements such as "the average family has 2.4 children." Here is another example:

By then, she will have shed 80 of the 240 pounds she weighed in with when she entered Peter Bent Brigham hospital obesity program. A third of her left behind! - The Boston Herald American, 7/7/77

The question seems to welcome references. There are certainly several that connect fractions with algebra. This paper,

Seigler et al. (2013), Fractions: the new frontier for theories of numerical development, Trends in Cognitive Sciences,

is a short survey of what is known and unknown about neural bases for one's knowledge of fractions. Whole number arithmetic knowledge has been studied, and the authors suggest that the representation of the knowledge fractions is an area ripe for investigation. It reviews (with references) why fractions are difficult and the relation of skill at fractions to skill at algebra. Generally - or, rather, I only know of papers that discuss the connection in that direction, with algebra skill being dependent on fractions skill. (OTOH, It's not widely read in this area.)

Id: 7831,

Score: 11,

is\_accepted\_answer: False,

language\_detection\_score: 0.9780

Subset: StackExchange

Figure 10: An example StackExchange document in MATHPILE . Here is a question from "matheducators" ".stackexchange.com" with two high-quality responses.

## D Details for Corpus Collection and Processing

The subjects from which we collected papers on arXiv are listed in Table 7. The specific StackExchange sites from which we gathered data are listed in Table 8. We illustrate the LaTeX display issue with an example in Figure 11.

During the collection process of arXiv, we undertook extensive transformations to enhance data clarity and consistency. Specifically, we (1) removed comments in each paper; (2) reverted many macro commands (e.g., “newcommand”) to their original forms; (3) omitted figure environments while retaining captions and figure labels; (4) excluded acknowledgements sections; (5) eliminated references in each paper; (6) condensed more than three consecutive empty lines to two; (7) replaced certain formatting commands like “hfill” and “vspace” with an empty line; (8) replaced the “maketitle” command in the main document body with the actual title (if available); (9) preserved only the content within the main body of the LaTeX document.

We summarize the parts of the dataset collection (cf. § 2) and global data preprocessing (cf. § 3) where human intervention was involved and whether the cleaning process was automated in the Table 9 and Table 10. We hope this provides a clearer understanding of MATHPILE construction process.

Table 7: The subject list during collecting corpus from arXiv.

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Subjects
math.AG, math.AT, math.AP, math.CT, math.CA, math.CO, math.AC, math.CV, math.DG, math.DS, math.FA, math.GM, math.GN, math.GT, math.GR, math.HO, math.IT, math.KT, math.LO, math.MP, math.MG, math.NT, math.NA, math.OA, math.OC, math.PR, math.QA, math.RT, math.RA, math.SP, math.ST, math.SG, math-ph, quant-ph, cs.CC, cs.CG, cs.DM, cs.DS, cs.FL, cs.GT, cs.LG, cs.NA, cs.LO, q-fin.MF, stat.CO, stat.ML, stat.ME, stat.OT, stat.TH, econ.TH

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Table 8: The site list during collecting corpus from StackExchange.

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Sites sourced from StackExchange
math.stackexchange.com, mathoverflow.net, mathematica.stackexchange.com, matheducators.stackexchange.com, hsm.stackexchange.com, physics.stackexchange.com, proofassistants.stackexchange.com, tex.stackexchange.com, datascience.stackexchange.com, cstheory.stackexchange.com, cs.stackexchange.com

---

Table 9: Details of Human Involvement and Automation in the MATHPILE Collection Process

MATHPILE Subset	Human Involvement in Data Collection	Cleaning Process Automated?
Textbooks	Manual search and download of open-source, free mathematics textbooks; quality check; setting cleaning rules	Yes, the automated application of the PDF conversion API and the document cleaning rules
arXiv Papers	Manual selection of relevant mathematical field categories; setting latex cleaning and formatting rules	Yes, automated cleaning steps like comment removal and format conversion
Wikipedia Mathematical Entries	Humans observed samples to define rules for cleaning irrelevant content such as copyright statements	Yes, automated HTML to Markdown conversion; and removal of extraneous lines
ProofWiki Entries	No significant human intervention (mainly data dump selection, reformatting design)	Yes, automated text parsing and formatting
StackExchange Discussions	Selection of relevant mathematics-related sites within the StackExchange network; setting filter thresholds	Yes, automated HTML parsing and conversion, score filtering
Common Crawl Web Pages	Manual adjustment of TF-IDF rules to improve mathematical content identification	Yes, automated application of rule-based mathematical document filtering

Table 10: Details of Human Involvement and Automation in the MATHPILE Global Data Processing Steps

Global Data Processing Step	Human Involvement	Cleaning Process Automated?
Language Identification	Manual adjustment of thresholds based on observed false positives	Yes, using FastText with custom thresholds
Data Cleaning & Filtering	Human observation to define rules for filtering irrelevant content	Yes, automated application of rules for content filtering and removal
Data Deduplication	Manual review of near-duplicate samples from different sources	Yes, automated using MinHash LSH
Data Contamination Detection	Human verification of flagged benchmark duplicates	Yes, automated detection based on pre-defined criteria

<pre># Abhyankar's inequality **Abhyankar's inequality** is an inequality involving extensions of valued fields in algebra, introduced by Abhyankar (1956). Abhyankar's inequality states that for an extension <math>K / k</math> of valued fields, the transcendence degree of <math>K / k</math> is at least the transcendence degree of the residue field extension plus the rank of the quotient of the valuation groups; here the rank of an abelian group <math>A</math> is defined as <math>\dim Q (A \otimes Q)</math>. <math>\{\mathbb{Q}\} (A \otimes \mathbb{Q})</math>.  ## References * Abhyankar, Shreeram (1956), "On the valuations centered in a local domain", <i>American Journal of Mathematics</i>, <b>78</b>(2): 321–348, doi:10.2307/2372519, ISSN 0002-9327, JSTOR 2372519, MR 0082477</pre>
<pre># Abhyankar's inequality **Abhyankar's inequality** is an inequality involving extensions of valued fields in algebra, introduced by Abhyankar (1956). Abhyankar's inequality states that for an extension <math>K / k</math> of valued fields, the transcendence degree of <math>K / k</math> is at least the transcendence degree of the residue field extension plus the rank of the quotient of the valuation groups; here the rank of an abelian group <math>A</math> is defined as <math>\dim Q (A \otimes Q)</math>. <math>\{\mathbb{Q}\} (A \otimes \mathbb{Q})</math>.  ## References * Abhyankar, Shreeram (1956), "On the valuations centered in a local domain", <i>American Journal of Mathematics</i>, <b>78</b>(2): 321–348, doi:10.2307/2372519, ISSN 0002-9327, JSTOR 2372519, MR 0082477</pre>

Figure 11: A document processed by html2text (above) compared to one obtained through another library Resiliparse plus DOM parsing (below).

## E Example for Quality Annotation

We present a cleaned example document with quality annotations (see Figure 12).

**A document from MATHPILE-Common Crawl**

**Text:**  
This number is called the Copeland–Erdős constant, and is known to be irrational and normal. I believe its transcendence or otherwise is an open problem. This source claims that it has been proved to be transcendental, but the paper they refer to is the one in which it was proved to be normal and so I think the source is mistaken.  
For now, the knowledge that it is almost surely transcendental will have to suffice!  
Not the answer you're looking for? Browse other questions tagged number-theory transcendental-numbers or ask your own question.  
Does the number 2.3, 5, 7, 11, 13 . . . exist and, if so, is it rational or irrational & or transcendental?  
Is 0.248163264128. . . a transcendental number?  
What is the name of this number? Is it transcendental?  
Is 0.112123123412345123456 . . . algebraic or transcendental?  
Is 0.121121111112111. . . a transcendental number?  
Do we know a transcendental number with a proven bounded continued fraction expansion?  
If we delete the non-primes from  $e$ , is the resulting number transcendental?  
Is there any known transcendental  $b$  such that  $b^b$  is also transcendental?  
...  
**Subset:** Common Crawl

---

**meta:**  
language\_detection\_score: 0.9118,  
char\_num\_after\_normalized: 887,  
contain\_at\_least\_two\_stop\_words: True,  
ellipsis\_line\_ratio: 0.0, idx: 95994,  
lines\_start\_with\_bullet\_point\_ratio: 0.0,  
mean\_length\_of\_alpha\_words: 4.2941,  
non\_alphabetical\_char\_ratio: 0.0234,  
symbols\_to\_words\_ratio: 0.0117,  
uppercase\_word\_ratio: 0.0117  
...

Figure 12: An example document after cleaning and filtering with quality annotations

## F Examples of Duplicates Encountered in the Deduplication Process

We provide some illustrative examples of duplicates from each source in the deduplication process, as shown in Table 11 to Table 16.

We also provide examples of downstream task benchmarks (i.e., MATH and MMLU-STEM) leaks identified during our data contamination detection process for our corpus (as shown in Table 17 and Table 18) and OpenWebMath (as shown in Table 19).

Table 11: Near-duplication matches found in CommonCrawl by MinHash LSH deduplication (in *italics*).

<p><i>In algebraic topology we often encounter chain complexes with extra multiplicative structure. For example, the cochain complex of a topological space has what is called the <math>E_\infty</math>-algebra structure which comes from the cup product.</i></p> <p><i>In this talk I present an idea for studying such chain complexes, <math>E_\infty</math> differential graded algebras (<math>E_\infty</math> DGAs), using stable homotopy theory. Namely, I discuss new equivalences between <math>E_\infty</math> DGAs that are defined using commutative ring spectra.</i></p> <p><b>ring spectra are equivalent. Quasi-isomorphic <math>E_\infty</math> DGAs are <math>E_\infty</math> topologically equivalent. However, the examples I am going to present show that the opposite is not true; there are <math>E_\infty</math> DGAs that are <math>E_\infty</math> topologically equivalent but not quasi-isomorphic. This says that between <math>E_\infty</math> DGAs, we have more equivalences than just the quasi-isomorphisms. I also discuss interaction of <math>E_\infty</math> topological equivalences with the Dyer-Lashof operations and cases where <math>E_\infty</math> topological equivalences and quasi-isomorphisms agree.</b></p>	<p><b>Özet :</b> <i>In algebraic topology we often encounter chain complexes with extra multiplicative structure. For example, the cochain complex of a topological space has what is called the <math>E_\infty</math>-algebra structure which comes from the cup product. In this talk I present an idea for studying such chain complexes, <math>E_\infty</math> differential graded algebras (<math>E_\infty</math> DGAs), using stable homotopy theory. Namely, I discuss new equivalences between <math>E_\infty</math> DGAs that are defined using commutative ring spectra. We say <math>E_\infty</math> DGAs are <math>E_\infty</math> topologically equivalent when the corresponding commutative ring spectra are equivalent. Quasi-isomorphic <math>E_\infty</math> DGAs are <math>E_\infty</math> topologically equivalent. However, the examples I am going to present show that the opposite is not true; there are <math>E_\infty</math> DGAs that are <math>E_\infty</math> topologically equivalent but not quasi-isomorphic. This says that between <math>E_\infty</math> DGAs, we have more equivalences than just the quasi-isomorphisms. I also discuss interaction of <math>E_\infty</math> topological equivalences with the Dyer-Lashof operations and cases where <math>E_\infty</math> topological equivalences and quasi-isomorphisms agree.</i></p>
<p>Université de la Saskatchewan, 1 - 4 juin 2015  <a href="http://www.smc.math.ca//2015f">www.smc.math.ca//2015f</a>          Comité d'organisation          Financement étudiants          Minisymposia invités          Minisymposia libres          Conférences libres          Horaire - Minisymposia invités          Open Problems          Graphs and matrices          Responsable et président: Shaun Fallat et Karen Meagher (University of Regina)  <i>WAYNE BARRETT, Brigham Young University</i>  <i>The Fiedler Vector and Tree Decompositions of Graphs [PDF]</i>  <i>In the 1970's Fiedler initiated a study of the second smallest eigenvalue of the Laplacian matrix <math>L</math> of a graph and the corresponding eigenvector(s). These "Fiedler" vectors have become spectacularly successful in revealing properties of the associated graph. A tree decomposition <math>\mathcal{T}</math> of a graph <math>G = (V, E)</math> is an associated tree whose nodes are subsets of <math>V</math> and whose edge set respects the structure of <math>G</math>. Tree decompositions have been used in the analysis of complex networks. This talk reports on an algorithm developed by students at BYU for obtaining a tree decomposition by means of Fiedler vector(s) of <math>G</math>.</i></p> <p>...</p> <p><i>Graphs that have a weighted adjacency matrix with spectrum <math>\{\lambda_1^{n-2}, \lambda_2^2\}</math> [PDF]</i>  <i>In this talk I will characterize the graphs which have an edge weighted adjacency matrix belonging to the class of <math>n \times n</math> involutions with spectrum equal to <math>\{\lambda_1^{n-2}, \lambda_2^2\}</math> for some <math>\lambda_1</math> and some <math>\lambda_2</math>. The connected graphs turn out to be the cographs constructed as the join of at least two unions of pairs of complete graphs, and possibly joined with one other complete graph.</i></p>	<p>University of Saskatchewan, June 1 - 4, 2015  <a href="http://www.cms.math.ca//2015">www.cms.math.ca//2015</a>          Invited Minisymposia          Contributed Minisymposia          Contributed Talks          Graphs and matrices          Organizer and Chair: Shaun Fallat and Karen Meagher (University of Regina)  <i>WAYNE BARRETT, Brigham Young University</i>  <i>The Fiedler Vector and Tree Decompositions of Graphs [PDF]</i>  <i>In the 1970's Fiedler initiated a study of the second smallest eigenvalue of the Laplacian matrix <math>L</math> of a graph and the corresponding eigenvector(s). These "Fiedler" vectors have become spectacularly successful in revealing properties of the associated graph. A tree decomposition <math>\mathcal{T}</math> of a graph <math>G = (V, E)</math> is an associated tree whose nodes are subsets of <math>V</math> and whose edge set respects the structure of <math>G</math>. Tree decompositions have been used in the analysis of complex networks. This talk reports on an algorithm developed by students at BYU for obtaining a tree decomposition by means of Fiedler vector(s) of <math>G</math>.</i></p> <p>...</p> <p><i>Graphs that have a weighted adjacency matrix with spectrum <math>\{\lambda_1^{n-2}, \lambda_2^2\}</math> [PDF]</i>  <i>In this talk I will characterize the graphs which have an edge weighted adjacency matrix belonging to the class of <math>n \times n</math> involutions with spectrum equal to <math>\{\lambda_1^{n-2}, \lambda_2^2\}</math> for some <math>\lambda_1</math> and some <math>\lambda_2</math>. The connected graphs turn out to be the cographs constructed as the join of at least two unions of pairs of complete graphs, and possibly joined with one other complete graph.</i></p>



Table 12: A near-duplication match found in arXiv by MinHashLSH deduplication (in *italics*).

<pre> \begin{document} \title{Querying Guarded Fragments via Resolution} \section{A detailed example}  Here we include some equations and theorem-like environments to show how these are labeled in a supplement and can be referenced from the main text. Consider the following equation: \begin{equation} \label{eq:suppa} a^2 + b^2 = c^2. \end{equation} You can also reference equations such as \cref{eq:matrices,eq:bb} from the main article in this supplement. \lipsum[100-101]  \begin{theorem} An example theorem. \end{theorem}  \lipsum[102]  \begin{lemma} An example lemma. \end{lemma}  \lipsum[103-105]  Here is an example citation: \cite{KoMa14}.  \section{Proof of Thm}[Proof of\cref{thm:bigthm}] \label{sec:proof}  \lipsum[106-112]  \section{Additional experimental results} \Cref{tab:foo} shows additional supporting evidence.  \begin{table}[htbp] \footnotesize \caption{Example table} \label{tab:foo} \begin{center} \begin{tabular}{ c c c } \hline Species &amp; \bf Mean &amp; \bf Std.~Dev. \\ \hline 1 &amp; 3.4 &amp; 1.2 \\ \hline 2 &amp; 5.4 &amp; 0.6 \\ \hline \end{tabular} \end{center} \end{table}  \end{document} </pre>	<pre> \begin{document} \title{Limited memory Kelley's Method Converges for Composite Convex and Submodular Objectives} \section{A detailed example}  Here we include some equations and theorem-like environments to show how these are labeled in a supplement and can be referenced from the main text. Consider the following equation: \begin{equation} \label{eq:suppa} a^2 + b^2 = c^2. \end{equation} You can also reference equations such as \cref{eq:matrices,eq:bb} from the main article in this supplement. \lipsum[100-101]  \begin{theorem} An example theorem. \end{theorem}  \lipsum[102]  \begin{lemma} An example lemma. \end{lemma}  \lipsum[103-105]  Here is an example citation: \cite{KoMa14}.  \section{Proof of Thm}[Proof of\cref{thm:bigthm}] \label{sec:proof}  \lipsum[106-112]  \section{Additional experimental results} \Cref{tab:foo} shows additional supporting evidence.  \begin{table}[htbp] \footnotesize \caption{Example table} \label{tab:foo} \begin{center} \begin{tabular}{ c c c } \hline Species &amp; \bf Mean &amp; \bf Std.~Dev. \\ \hline 1 &amp; 3.4 &amp; 1.2 \\ \hline 2 &amp; 5.4 &amp; 0.6 \\ \hline \end{tabular} \end{center} \end{table}  \end{document} </pre>
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---

```
\section{Definition:Constructed
Semantics/Instance 4/Rule of
Idempotence}
Tags: Formal Semantics
```

```
\begin{theorem}
The Rule of Idempotence:
:$(p \lor p) \implies p$
is a tautology in Instance 4 of
constructed semantics.
\end{theorem}
```

```
\begin{proof}
By the definitional abbreviation for
the conditional:
: $\mathbf{A} \implies \mathbf{B} =_{\text{def}} \neg \mathbf{A} \lor \mathbf{B}$
```

```
the Rule of Idempotence can be
written as:
: $\neg \left( (p \lor p) \right) \lor p$
```

```
This evaluates as follows:
: $\begin{array}{|cccc|c|c|} \hline
\neg & \& (p & \& \lor & \& p) & \& \lor & \& p \\ \hline
1 & \& 0 & \& 0 & \& 0 & \& 0 & \& 0 \\ \hline
0 & \& 1 & \& 1 & \& 1 & \& 0 & \& 1 \\ \hline
0 & \& 2 & \& 2 & \& 2 & \& 0 & \& 2 \\ \hline
2 & \& 3 & \& 3 & \& 3 & \& 0 & \& 3 \\ \hline
\end{array}$
```

```
\end{array}$
{{qed}}
Category: Formal Semantics
\end{proof}
```

---

```
\section{Definition:Constructed
Semantics/Instance 5/Rule of
Idempotence}
Tags: Formal Semantics
```

```
\begin{theorem}
The Rule of Idempotence:
:$(p \lor p) \implies p$
is a tautology in Instance 5 of
constructed semantics.
\end{theorem}
```

```
\begin{proof}
By the definitional abbreviation for
the conditional:
: $\mathbf{A} \implies \mathbf{B} =_{\text{def}} \neg \mathbf{A} \lor \mathbf{B}$
```

```
the Rule of Idempotence can be
written as:
: $\neg \left( (p \lor p) \right) \lor p$
```

```
This evaluates as follows:
: $\begin{array}{|cccc|c|c|} \hline
\neg & \& (p & \& \lor & \& p) & \& \lor & \& p \\ \hline
1 & \& 0 & \& 0 & \& 0 & \& 0 & \& 0 \\ \hline
0 & \& 1 & \& 1 & \& 1 & \& 0 & \& 1 \\ \hline
3 & \& 2 & \& 2 & \& 2 & \& 0 & \& 2 \\ \hline
0 & \& 3 & \& 3 & \& 3 & \& 0 & \& 3 \\ \hline
\end{array}$
```

```
\end{array}$
{{qed}}
Category: Formal Semantics
\end{proof}
```

---

```
\section{Imaginary Part of Complex
Product}
Tags: Complex Multiplication
```

```
\begin{theorem}
Let $z_1$ and $z_2$ be complex
numbers.
Then:
: $\map \Im \{z_1 z_2\} = \map \Re \{z_1\} \map \Im \{z_2\} + \map \Im \{z_1\} \map \Re \{z_2\}$
\end{theorem}
```

```
\begin{proof}
Let $z_1 = x_1 + i y_1$ and $z_2 = x_2 + i y_2$.
By definition of complex
multiplication:
: $z_1 z_2 = x_1 x_2 - y_1 y_2 + i \paren{x_1 y_2 + x_2 y_1}$
Then
{{begin-eqn}}
{{eqn | l = \map \Im \{z_1 z_2\}
| r = x_1 y_2 + x_2 y_1
| c = {{Defof|Imaginary Part}}
}}
{{eqn | r = \map \Re \{z_1\} \map \Im \{z_2\} + \map \Im \{z_1\} \map \Re \{z_2\}
| c = {{Defof|Imaginary Part}}
}}
{{end-eqn}}
{{qed}}
\end{proof}
```

---

```
\section{Real Part of Complex
Product}
Tags: Complex Multiplication
```

```
\begin{theorem}
Let $z_1$ and $z_2$ be complex
numbers.
Then:
: $\map \Re \{z_1 z_2\} = \map \Re \{z_1\} \map \Re \{z_2\} - \map \Im \{z_1\} \map \Im \{z_2\}$
\end{theorem}
```

```
\begin{proof}
Let $z_1 = x_1 + i y_1$ and $z_2 = x_2 + i y_2$.
By definition of complex
multiplication:
: $z_1 z_2 = x_1 x_2 - y_1 y_2 + i \paren{x_1 y_2 + x_2 y_1}$
Then:
{{begin-eqn}}
{{eqn | l = \map \Re \{z_1 z_2\}
| r = x_1 x_2 - y_1 y_2
| c = {{Defof|Real Part}}
}}
{{eqn | r = \map \Re \{z_1\} \map \Re \{z_2\} - \map \Im \{z_1\} \map \Im \{z_2\}
| c = {{Defof|Real Part}}
}}
{{end-eqn}}
{{qed}}
\end{proof}
```

Table 14: Duplication matches found in Wikipedia by MinHash LSH deduplication (in *italics*).

# HP-42S	# HP-42S
<p><i>The **HP-42S RPN Scientific** is a programmable RPN Scientific hand held calculator introduced by Hewlett-Packard in 1988. It has advanced functions suitable for applications in mathematics, linear algebra, statistical analysis, computer science and others.</i></p>	<p><i>The **HP-42S RPN Scientific** is a programmable RPN Scientific hand held calculator introduced by Hewlett-Packard in 1988. It has advanced functions suitable for applications in mathematics, linear algebra, statistical analysis, computer science and others.</i></p>
<p><i>HP-42S</i>  <i>The HP-42S</i></p> <hr/> <p>Type  Programmable scientific  Manufacturer  Hewlett-Packard  Introduced  1988  Discontinued  1995 Calculator  Entry model  RPN  Precision  12 display digits (15 digits internally),[1] exponent <math>\pm 499</math>  Display type  LCD dot-matrix  Display size  2 lines, 22 characters, 131×16 pixels CPU  Processor  Saturn (Lewis) Programming  Programming language(s)  RPN key stroke (fully merged)  Firmware memory  64 KB of ROM  Program steps  7200 Interfaces  Ports  IR (Infrared) printing Other  Power supply  3×1.5V button cell batteries (Panasonic LR44, Duracell PX76A/675A or Energizer 357/303)  Weight  6 oz (170 g)  Dimensions  148×80×15mm</p> <p>## Overview</p> <p><i>Perhaps the HP-42S was to be released as a replacement for the aging HP-41 series as it is designed to be compatible with all programs written for the HP-41. Since it lacked expandability, and lacked any real I/O ability, both key features of the HP-41 series, it was marketed as an HP-15C replacement.</i></p> <p><i>The 42S, however, has a much smaller form factor than the 41, and features many more built-in functions, such as a matrix editor, complex number support, an equation solver, user-defined menus, and basic graphing capabilities (the 42S can draw graphs only by programs). Additionally, it features a two-line dot matrix display, which made stack manipulation easier to understand.</i></p> <p><i>Production of the 42S ended in 1995.[2] As this calculator is regarded amongst the best ever made in terms of quality, key stroke feel, ease of programming, and daily usability for engineers,[3] in the HP calculator community the 42S has become famous for its high prices in online auctions, up to several times its introduction price, which has created a scarcity for utility end users.</i></p>	<p><i>HP-42S</i>  <i>The HP-42S</i></p> <hr/> <p>Type  Programmable scientific  Manufacturer  Hewlett-Packard  Introduced  1988  Discontinued  1995 Calculator  Entry model  RPN  Precision  12 display digits (15 digits internally),[1] exponent <math>\pm 499</math>  Display type  LCD dot-matrix  Display size  2 lines, 22 characters, 131×16 pixels CPU  Processor  Saturn (Lewis) Programming  Programming language(s)  RPN key stroke (fully merged)  Firmware memory  64 KB of ROM  Program steps  7200 Interfaces  Ports  IR (Infrared) printing Other  Power supply  3×1.5V button cell batteries (Panasonic LR44, Duracell PX76A/675A or Energizer 357/303)  Weight  6 oz (170 g)  Dimensions  148×80×15mm</p> <p>## Overview</p> <p><i>Perhaps the HP-42S was to be released as a replacement for the aging HP-41 series as it is designed to be compatible with all programs written for the HP-41. Since it lacked expandability, and lacked any real I/O ability, both key features of the HP-41 series, it was marketed as an HP-15C replacement.</i></p> <p><i>The 42S, however, has a much smaller form factor than the 41, and features many more built-in functions, such as a matrix editor, complex number support, an equation solver, user-defined menus, and basic graphing capabilities (the 42S can draw graphs only by programs). Additionally, it features a two-line dot matrix display, which made stack manipulation easier to understand.</i></p> <p><i>Production of the 42S ended in 1995.[2] As this calculator is regarded amongst the best ever made in terms of quality, key stroke feel, ease of programming, and daily usability for engineers,[3] in the HP calculator community the 42S has become famous for its high prices in online auctions, up to several times its introduction price, which has created a scarcity for utility end users.</i></p>

**Table 15: Duplication matches found in Textbooks by MinHash LSH deduplication (in *italics*).**

<p><i># Basic Concepts in Graph Theory</i></p> <p><i>## Section 1: What is a Graph?</i></p> <p><i>There are various types of graphs, each with its own definition. Unfortunately, some people apply the term "graph" rather loosely, so you can't be sure what type of graph they're talking about unless you ask them. After you have finished this chapter, we expect you to use the terminology carefully, not loosely. To motivate the various definitions, we'll begin with some examples.</i></p> <p><i>Example 1 (A computer network) Computers are often linked with one another so that they can interchange information. Given a collection of computers, we would like to describe this linkage in fairly clean terms so that we can answer questions such as "How can we send a message from computer A to computer B using the fewest possible intermediate computers?"</i></p> <p><i>We could do this by making a list that consists of pairs of computers that are connected. Note that these pairs are unordered since, if computer C can communicate with computer D, then the reverse is also true. (There are sometimes exceptions to this, but they are rare and we will assume that our collection of computers does not have such an exception.) Also, note that we have implicitly assumed that the computers are distinguished from each other: It is insufficient to say that "A PC is connected to a Mac." We must specify which PC and which Mac. Thus, each computer has a unique identifying label of some sort.</i></p> <p><i>For people who like pictures rather than lists, we can put dots on a piece of paper, one for each computer. We label each dot with a computer's identifying label and draw a curve connecting two dots if and only if the corresponding computers are connected. Note that the shape of the curve does not matter (it could be a straight line or something more complicated) because we are only interested in whether two computers are connected or not. Below are two such pictures of the same graph. Each computer has been labeled by the initials of its owner.</i></p> <p><i>...</i></p> <p><i>## Basic Concepts in Graph Theory</i></p> <p><i>The notation <math>\mathcal{P}_k(V)</math> stands for the set of all <math>k</math>-element subsets of the set <math>V</math>. Based on the previous example we have</i></p> <p><i>Definition 1 (Simple graph) A simple graph <math>G</math> is a pair <math>G = (V, E)</math> where</i></p> <ul style="list-style-type: none"> <li><i>- <math>V</math> is a finite set, called the vertices of <math>G</math>, and</i></li> <li><i>- <math>E</math> is a subset of <math>\mathcal{P}_2(V)</math> (i.e., a set <math>E</math> of two-element subsets of <math>V</math>), called the edges of <math>G</math>.</i></li> </ul> <p><i>...</i></p>	<p><i># Basic Concepts in Graph Theory</i></p> <p><i>## Section 1: What is a Graph?</i></p> <p><i>There are various types of graphs, each with its own definition. Unfortunately, some people apply the term "graph" rather loosely, so you can't be sure what type of graph they're talking about unless you ask them. After you have finished this chapter, we expect you to use the terminology carefully, not loosely. To motivate the various definitions, we'll begin with some examples.</i></p> <p><i>Example 1 (A computer network) Computers are often linked with one another so that they can interchange information. Given a collection of computers, we would like to describe this linkage in fairly clean terms so that we can answer questions such as "How can we send a message from computer A to computer B using the fewest possible intermediate computers?"</i></p> <p><i>We could do this by making a list that consists of pairs of computers that are connected. Note that these pairs are unordered since, if computer C can communicate with computer D, then the reverse is also true. (There are sometimes exceptions to this, but they are rare and we will assume that our collection of computers does not have such an exception.) Also, note that we have implicitly assumed that the computers are distinguished from each other: It is insufficient to say that "A PC is connected to a Mac." We must specify which PC and which Mac. Thus, each computer has a unique identifying label of some sort.</i></p> <p><i>For people who like pictures rather than lists, we can put dots on a piece of paper, one for each computer. We label each dot with a computer's identifying label and draw a curve connecting two dots if and only if the corresponding computers are connected. Note that the shape of the curve does not matter (it could be a straight line or something more complicated) because we are only interested in whether two computers are connected or not. Below are two such pictures of the same graph. Each computer has been labeled by the initials of its owner.</i></p> <p><i>...</i></p> <p><i>## Basic Concepts in Graph Theory</i></p> <p><i>The notation <math>\mathcal{P}_k(V)</math> stands for the set of all <math>k</math>-element subsets of the set <math>V</math>. Based on the previous example we have</i></p> <p><i>Definition 1 (Simple graph) A simple graph <math>G</math> is a pair <math>G = (V, E)</math> where</i></p> <ul style="list-style-type: none"> <li><i>- <math>V</math> is a finite set, called the vertices of <math>G</math>, and</i></li> <li><i>- <math>E</math> is a subset of <math>\mathcal{P}_2(V)</math> (i.e., a set <math>E</math> of two-element subsets of <math>V</math>), called the edges of <math>G</math>.</i></li> </ul> <p><i>...</i></p>
---	---

Table 16: Near-duplication matches found in StackExchange by MinHash LSH deduplication (in *italics*).

<p>This was originally posted on mathoverflow, but it seems it's more appropriate to post here.</p> <p><i>Let <math>B</math> be a paracompact space with the property that any (topological) vector bundle <math>E \rightarrow B</math> is trivial. What are some non-trivial examples of such spaces, and are there any interesting properties that characterize them?</i></p> <p><i>For simple known examples we of course have contractible spaces, as well as the 3-sphere <math>S^3</math>. This one follows from the fact that its rank <math>n</math> vector bundles are classified by <math>\pi_3(BO(n)) = \pi_2(O(n)) = 0</math>. I'm primarily interested in the case where <math>B</math> is a closed manifold. Do we know any other such examples?</i></p> <p><i>There is this nice answer to a MSE question which talks about using the Whitehead tower of the appropriate classifying space to determine whether a bundle is trivial or not. This seems like a nice tool (of which I am not familiar with) to approaching this problem. As a secondary question, could I ask for some insight/references to this approach?</i></p> <p><i>EDIT Now that we know from the answer all the examples for closed 3-manifolds (integral homology spheres), I guess I can now update the question to the case of higher odd dimensions. Does there exist a higher dimensional example?</i></p>	<p><i>Let <math>B</math> be a paracompact space with the property that any (topological) vector bundle <math>E \rightarrow B</math> is trivial. What are some non-trivial examples of such spaces, and are there any interesting properties that characterize them?</i></p> <p><i>For simple known examples we of course have contractible spaces, as well as the 3-sphere <math>S^3</math>. This one follows from the fact that its rank <math>n</math> vector bundles are classified by <math>\pi_3(BO(n)) = \pi_2(O(n)) = 0</math>. I'm primarily interested in the case where <math>B</math> is a closed manifold. Do we know any other such examples?</i></p> <p><i>There is this nice answer to a MSE question which talks about using the Whitehead tower of the appropriate classifying space to determine whether a bundle is trivial or not. This seems like a nice tool (of which I am not familiar with) to approaching this problem. As a secondary question, could I ask for some insight/references to this approach?</i></p> <p><i>EDIT Now that we know from the answers all the examples for closed 3-manifolds, I guess I can now update the question to the case of higher odd dimensions. Does there exist a higher dimensional example?</i></p>
<p>This is a copy of my question on MSE (<a href="https://math.stackexchange.com/questions/3372432">https://math.stackexchange.com/questions/3372432</a>) because this forum seems better suited for historical questions:</p> <p><i>In 1985, Gosper used the not-yet-proven formula by Ramanujan</i></p> $\frac{1}{\pi} = \frac{2\sqrt{2}}{99^2} \cdot \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \cdot \frac{26390n + 1103}{396^{4n}}$ <p><i>to compute <math>17 \cdot 10^6</math> digits of <math>\pi</math>, at that time a new world record.</i></p> <p><i>Here (<a href="https://www.cs.princeton.edu/courses/archive/fall98/cs126/refs/pi-ref.txt">https://www.cs.princeton.edu/courses/archive/fall98/cs126/refs/pi-ref.txt</a>) it reads:</i></p> <p><i>There were a few interesting things about Gosper's computation. First, when he decided to use that particular formula, there was no proof that it actually converged to <math>\pi</math>! Ramanujan never gave the math behind his work, and the Borweins had not yet been able to prove it, because there was some very heavy math that needed to be worked through. It appears that Ramanujan simply observed the equations were converging to the 1103 in the formula, and then assumed it must actually be 1103. (Ramanujan was not known for rigor in his math, or for providing any proofs or intermediate math in his formulas.) The math of the Borwein's proof was such that after he had computed 10 million digits, and verified them against a known calculation, his computation became part of the proof. Basically it was like, if you have two integers differing by less than one, then they have to be the same integer.</i></p> <p><i>Now my historical question: Who was the first to prove this formula? Was it Gosper because he added the last piece of the proof, or was it the Borweins, afterwards? And was Gosper aware of this proof when he did his computation?</i></p>	<p><i>In 1985, Gosper used the not-yet-proven formula by Ramanujan</i></p> $\frac{1}{\pi} = \frac{2\sqrt{2}}{99^2} \cdot \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \cdot \frac{26390n + 1103}{99^{4n}}$ <p><i>to compute <math>17 \cdot 10^6</math> digits of <math>\pi</math>, at that time a new world record.</i></p> <p><i>Here (<a href="https://www.cs.princeton.edu/courses/archive/fall98/cs126/refs/pi-ref.txt">https://www.cs.princeton.edu/courses/archive/fall98/cs126/refs/pi-ref.txt</a>) it reads:</i></p> <p><i>There were a few interesting things about Gosper's computation. First, when he decided to use that particular formula, there was no proof that it actually converged to <math>\pi</math>! Ramanujan never gave the math behind his work, and the Borweins had not yet been able to prove it, because there was some very heavy math that needed to be worked through. It appears that Ramanujan simply observed the equations were converging to the 1103 in the formula, and then assumed it must actually be 1103. (Ramanujan was not known for rigor in his math, or for providing any proofs or intermediate math in his formulas.) The math of the Borwein's proof was such that after he had computed 10 million digits, and verified them against a known calculation, his computation became part of the proof. Basically it was like, if you have two integers differing by less than one, then they have to be the same integer.</i></p> <p><i>Now my historical question: Who was the first to prove this formula? Was it Gosper because he added the last piece of the proof, or was it the Borweins, afterwards? And was Gosper aware of this proof when he did his computation?</i></p>

Table 17: Exact match examples from the test set of MATH benchmark found in Textbooks by line-level exact match deduplication (in *italics*).

---

*Coin A is flipped three times and coin B is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?*

Video Solution

Answer:

## Problem 3.2.2 (AMC 10)

Two tour guides are leading six tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the stipulation that each guide must take at least one tourist. How many different groupings of guides and tourists are possible?

.....

*One morning each member of Angela's family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?*

Answer:

## Problem 20.2.15 (AMC 12)

The state income tax where Kristin lives is levied at the rate of  $p\%$  of the first \$28000 of annual income plus  $(p + 2)\%$  of any amount above \$28000. Kristin noticed that the state income tax she paid amounted to  $(p + 0.25)\%$  of her annual income. What was her annual income?

Answer:

.....

*Find the least positive integer  $k$  for which the equation  $\lfloor \frac{2002}{n} \rfloor = k$  has no integer solutions for  $n$ . (The notation  $\lfloor x \rfloor$  means the greatest integer less than or equal to  $x$ .)*

Answer:

## Problem 40.1.9 (AIME)

Find the number of positive integers  $n$  less than 1000 for which there exists a positive real number  $x$  such that  $n = x \lfloor x \rfloor$ .', ', 'Note:  $\lfloor x \rfloor$  is the greatest integer less than or equal to  $x$ .'

.....

*What is the sum of the roots of  $z^{12} = 64$  that have a positive real part?*

Answer:

## Problem 45.8.13 (AMC 12)

The complex numbers  $z$  and  $w$  satisfy  $z^{13} = w$ ,  $w^{11} = z$ , and the imaginary part of  $z$  is  $\sin \frac{m\pi}{n}$ , for relatively prime positive integers  $m$  and  $n$  with  $m < n$ . Find  $n$ .'

Answer:

.....

Table 18: Exact match examples from the test set of MATH benchmark found in CommonCrawl by line-level exact match deduplication (in *italics*). In these examples, we only observe repeated questions from MATH, but do not identify duplicate answers.

*Let  $x$  and  $y$  be real numbers satisfying  $x^4y^5 + y^4x^5 = 810$  and  $x^3y^6 + y^3x^6 = 945$ . Evaluate  $2x^3 + (xy)^3 + 2y^3$ .*  
 Let  $x_1 < x_2 < x_3$  be the three real roots of the equation  $\sqrt{2014x^3 - 4029x^2 + 2} = 0$ . Find  $x_2(x_1 + x_3)$ .  
 Let  $m$  be the largest real solution to the equation

$$\frac{3}{x-3} + \frac{5}{x-5} + \frac{17}{x-17} + \frac{19}{x-19} = x^2 - 11x - 4$$

There are positive integers  $a, b,$  and  $c$  such that  $m = a + \sqrt{b + \sqrt{c}}$ . Find  $a + b + c$ .  
 Let  $f(x) = x^4 + ax^3 + bx^2 + cx + d$ . If  $f(-1) = -1, f(2) = -4, f(-3) = -9,$  and  $f(4) = -16$ . Find  $f(1)$ .  
 Solve in positive integers  $x^2 - 4xy + 5y^2 = 169$ .  
 Solve in integers the question  $x + y = x^2 - xy + y^2$ .  
 Solve in integers  $\frac{x+y}{x^2-xy+y^2} = \frac{3}{7}$ .  
 Prove the product of 4 consecutive positive integers is a perfect square minus 1.  
 For any arithmetic sequence whose terms are all positive integers, show that if one term is a perfect square, this sequence must have infinite number of terms which are perfect squares.  
 Prove there exist infinite number of positive integer  $a$  such that for any positive integer  $n, n^4 + a$  is not a prime number.

.....

*The real root of the equation  $8x^3 - 3x^2 - 3x - 1 = 0$  can be written in the form  $\frac{\sqrt[3]{a} + \sqrt[3]{b+1}}{c}$ , where  $a, b,$  and  $c$  are positive integers. Find  $a + b + c$ .*

Find the number of positive integers  $m$  for which there exist nonnegative integers  $x_0, x_1, \dots, x_{2011}$  such that

$$m^{x_0} = \sum_{k=1}^{2011} m^{x_k}.$$

Suppose  $x$  is in the interval  $[0, \frac{\pi}{2}]$  and  $\log_{24 \sin x}(24 \cos x) = \frac{3}{2}$ . Find  $24 \cot^2 x$ .  
 Let  $P(x)$  be a quadratic polynomial with real coefficients satisfying  $x^2 - 2x + 2 \leq P(x) \leq 2x^2 - 4x + 3$  for all real numbers  $x$ , and suppose  $P(11) = 181$ . Find  $P(16)$ .  
 Let  $(a, b, c)$  be the real solution of the system of equations  $x^3 - xyz = 2, y^3 - xyz = 6, z^3 - xyz = 20$ . The greatest possible value of  $a^3 + b^3 + c^3$  can be written in the form  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .  
 Find the smallest positive integer  $n$  with the property that the polynomial  $x^4 - nx + 63$  can be written as a product of two nonconstant polynomials with integer coefficients.  
 The zeros of the function  $f(x) = x^2 - ax + 2a$  are integers. What is the sum of the possible values of  $a$ ?  
 Let  $a, b,$  and  $c$  be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation  $(x-a)(x-b) + (x-b)(x-c) = 0$ ?  
 At the theater children get in for half price. The price for 5 adult tickets and 4 child tickets is 24.50. How much would 8 adult tickets and 6 child tickets cost?  
 The quadratic equation  $x^2 + px + 2p = 0$  has solutions  $x = a$  and  $x = b$ . If the quadratic equation  $x^2 + cx + d = 0$  has solutions  $x = a + 2$  and  $x = b + 2$ , what is the value of  $d$ ?

.....

*Find the smallest positive integer  $n$  with the property that the polynomial  $x^4 - nx + 63$  can be written as a product of two nonconstant polynomials with integer coefficients.*

The zeros of the function  $f(x) = x^2 - ax + 2a$  are integers. What is the sum of the possible values of  $a$ ?  
 Let  $a, b,$  and  $c$  be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation  $(x-a)(x-b) + (x-b)(x-c) = 0$ ?  
 At the theater children get in for half price. The price for 5 adult tickets and 4 child tickets is 24.50. How much would 8 adult tickets and 6 child tickets cost?  
 The quadratic equation  $x^2 + px + 2p = 0$  has solutions  $x = a$  and  $x = b$ . If the quadratic equation  $x^2 + cx + d = 0$  has solutions  $x = a + 2$  and  $x = b + 2$ , what is the value of  $d$ ?

PolynomialAndEquation Root Delta SpecialEquation Function NumberTheoryBasic IndeterminateEquation SqueezeMethod Pythagore anTripletFormula TrigIdentity Inequality LogicalAndReasoning  
 AMC10/12 AIME IMO

US International

With Solutions

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.....

Table 19: Exact match examples from the test set of MATH benchmark (upper) and MMLU-STEM (bottom) found in OpenWebMath by line-level exact match deduplication (in *italics*). In these examples, we only observe repeated questions, but do not identify duplicate answers.

---

*The sum of an infinite geometric series is a positive number  $S$ , and the second term in the series is 1. What is the smallest possible value of  $S$ ?*

- (A)  $\frac{1+\sqrt{5}}{2}$     (B) 2    (C)  $\sqrt{5}$     (D) 3    (E) 4

## Problem 17

All the numbers 2, 3, 4, 5, 6, 7 are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

- (A) 312    (B) 343    (C) 625    (D) 729    (E) 1680

...

*What is the value of  $b + c$  if  $x^2 + bx + c > 0$  only when  $x \in (-\infty, -2) \cup (3, \infty)$ ?*

May 11, 2020

...

*An ambulance travels at 40 mph and can follow a 20-mile route making no stops to get to the hospital. A helicopter travels at one mile per minute, and the air route is 15 miles to get to the same hospital. However, the helicopter takes three minutes for takeoff and three minutes for landing. How many fewer minutes does it take for the helicopter to complete its trip (takeoff, flight and landing) than for the ambulance to complete its trip?*

Apr 6, 2020

#1  
+34  
0

Keep in mind that Time=Distance/Speed

---

*What is the greatest possible area of a triangular region with one vertex at the center of a circle of radius 1 and the other two vertices on the circle?*

A bad first step is to put the center at the origin, one point at (1,0), and one point at (sin x, cos x).

A start is the area of a triangle with included angle expression

$$\frac{a \times b \times \sin \theta}{2}$$

Assuming  $\theta$  in radians. If theta is  $\pi/2$  then we have a right triangle. Let a=b=1. Area expression is

$$A = (\sin \theta)/2$$

This is maximum for  $\theta = \pi/2$ .

Answer is maximum area for a right triangle.

...

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## G Evaluation of Continual Pre-trained Models on General Language Benchmarks

Does continual pre-training on MATHPILE lead to improvements in general language benchmarks? To explore this, we evaluated some continual pre-trained models on MATHPILE (in Table 4) in several representative benchmarks including PIQA [8], ARC-Easy [15], ARC-Challenge [15], SciQ [64] and HellaSwag [69]. For these evaluations, we used the infrastructure provided by EleutherAI’s lm-evaluation-harness,<sup>7</sup> and following common practices, we report the accuracy (acc norm) metric.

Table 20: Performance of continual pre-trained models on general language benchmarks

<b>Models</b>	<b>PiQA</b>	<b>ARC-Challenge</b>	<b>ARC Easy</b>	<b>SciQ</b>	<b>Hallswag</b>
Mistral-7B	81.93	53.75	79.58	94.0	81.05
+ Textbooks	80.14	52.73	79.92	95.6	81.15
+ Wikipedia	80.57	56.48	79.71	94.8	82.07
+ Stackexchange	80.41	49.66	75.38	90.5	82.87

One important point that needs to be emphasised is that when enhancing a model’s capabilities in a specific domain, it is typically necessary to mix the new domain-specific training data with the original training data, which helps to prevent catastrophic forgetting. In current experiments, we conducted continual pre-training exclusively on math-specific data, which means that we did not necessarily expect improvements in general language modeling benchmarks and, in some cases, a regression could occur. As shown in Table 20, after continual pre-training on MATHPILE subsets, the model’s general language modeling abilities did not experience significant degradation. In fact, there were some improvements on certain benchmarks, though some metrics did see slight declines.

<sup>7</sup><https://github.com/EleutherAI/lm-evaluation-harness>

## Checklist

1. For all authors...
  - (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
  - (b) Did you describe the limitations of your work? [Yes]
  - (c) Did you discuss any potential negative societal impacts of your work? [No]
  - (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
  - (a) Did you state the full set of assumptions of all theoretical results? [N/A]
  - (b) Did you include complete proofs of all theoretical results? [N/A]
3. If you ran experiments (e.g. for benchmarks)...
  - (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes]
  - (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes]
  - (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [No]
  - (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes]
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
  - (a) If your work uses existing assets, did you cite the creators? [Yes]
  - (b) Did you mention the license of the assets? [Yes]
  - (c) Did you include any new assets either in the supplemental material or as a URL? [No]
  - (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [Yes]
  - (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [Yes]
5. If you used crowdsourcing or conducted research with human subjects...
  - (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
  - (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
  - (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]