A Appendix

A.1 Proofs

In this section we restate and provide proofs of the statements made in the main text.

**Property 1.** For any \( \Pi \subseteq \Pi, \nu \subseteq \nu \) and \( M \subseteq M \), it follows that \( M^k(\Pi, \nu; 0) = M^k(\Pi, \nu) ) \) and \( M^{\infty}(\Pi; 0) = M^{\infty}(\Pi) \).

**Proof.** Any \( \tilde{m} \in M^k(\Pi, \nu; 0) \) satisfies \( \|\tilde{T}^k_{\pi} v - T^k_{\pi} v\| = 0 \) \( \forall \pi \in \Pi, \forall v \in \nu \). Similarly any \( \tilde{m} \in M^k(\Pi, \nu) \) satisfies analogous equality constraints \( T^k_{\pi} v = T^k_{\pi} v \) \( \forall \pi \in \Pi, \forall v \in \nu \). Since \( \|\cdot\| \) is a norm, we know that \( \|\tilde{T}^k_{\pi} v - T^k_{\pi} v\| = 0 \iff T^k_{\pi} v = T^k_{\pi} v \), hence \( M^k(\Pi, \nu; 0) = M^k(\Pi, \nu) \). The same logic applies to APVE classes.

**Property 2.** For any \( \epsilon \in \mathbb{R}^+, M \subseteq M \subseteq M, \Pi \subseteq \Pi' \subseteq \Pi \) and \( \nu \subseteq \nu' \subseteq \nu \), it follows that

\[
M^k(\Pi', \nu'; \epsilon) \subseteq M^k(\Pi, \nu; \epsilon) \subseteq M^k(\Pi, \nu; \epsilon).
\]

**Proof.** An AVE class, \( M^k(\Pi', \nu'; \epsilon) \) satisfies a series of constraints of the form \( \|\tilde{T}^k_{\pi} v - T^k_{\pi} v\| \leq \epsilon \) for each pair of \( \pi, v \in \Pi' \times \nu' \). Considering another pair of sub-sets \( \Pi \subseteq \Pi' \) and \( \nu \subseteq \nu' \), we can partition the first pair as follows:

\[
\Pi' \times \nu' = (\Pi' \setminus \Pi \times \nu') \cup (\Pi' \setminus \Pi \times \nu) \cup (\Pi \times \nu' \setminus \nu) \cup (\Pi \times \nu)
\]

Accordingly,

\[
M^k(\Pi', \nu'; \epsilon) = M^k(\Pi' \setminus \Pi, \nu' \setminus \nu; \epsilon) \cap M^k(\Pi' \setminus \Pi, \nu \setminus \nu; \epsilon) \cap M^k(\Pi, \nu' \setminus \nu; \epsilon) \cap M^k(\Pi, \nu; \epsilon)
\]

satisfying the first subset relation in Eq. 7. For the next subset relation, we simply note that

\[
M^k(\Pi, \nu; \epsilon) = (M \setminus M) (\Pi, \nu; \epsilon) \cup M^k(\Pi, \nu; \epsilon) \subseteq M^k(\Pi, \nu; \epsilon),
\]

completing the proof.

**Property 3.** For any \( \Pi \subseteq \Pi, \nu \subseteq \nu \) and \( \epsilon, \epsilon' \in \mathbb{R}^+ \) such that \( \epsilon' \geq \epsilon \), it follows that

\[
M^k(\Pi, \nu; \epsilon) \subseteq M^k(\Pi, \nu; \epsilon').
\]

**Proof.** For any \( \tilde{m} \in M^k(\Pi, \nu; \epsilon) \) a number of AVE constraints are respected: \( \|\tilde{T}_{\pi}^k v - T_{\pi}^k v\| \leq \epsilon \) for each pair \( \pi, v \in \Pi \times \nu \). Since \( \epsilon' \geq \epsilon \), it follows that \( \|\tilde{T}_{\pi}^k v - T_{\pi}^k v\| \leq \epsilon \leq \epsilon' \) as well and hence \( \tilde{m} \in M^k(\Pi, \nu; \epsilon') \). Thus \( M^k(\Pi, \nu; \epsilon) \subseteq M^k(\Pi, \nu; \epsilon') \) as needed.

**Proposition 1.** For any \( \epsilon \in \mathbb{R}^+, \Pi, \Pi' \subseteq \Pi, \nu, \nu' \subseteq \nu \) and \( K \in \mathbb{Z}^+ \), there exists some \( \epsilon' \in \mathbb{R}^+ \) such that

\[
M^k(\Pi, \nu; \epsilon) \subseteq M^k(\Pi', \nu'; \epsilon').
\]

Moreover, if \( M, \nu \) and \( \nu' \) are bounded then \( \epsilon' \) is finite.

**Proof.** Denote \( v_{\max} = \max_{s \in S, v \in \nu} v(s), \tilde{r}_{\max} = \max_{s \in S, a \in A, \tilde{m} \in M} r(s, a) \) and consider any \( \tilde{m} \in M^k(\Pi, \nu; \epsilon) \). We can then write

\[
\|\tilde{T}_{\pi}^k v - T_{\pi}^k v\| \leq \max_{s} |\tilde{T}_{\pi}^k v(s)| + \max_{s} |T_{\pi}^k v(s)| \leq 2 \max\{\tilde{r}_{\max}, r_{\max}\} \frac{1-\gamma^K}{1-\gamma} + \gamma^K v_{\max}
\]

for any \( \pi \in \Pi', v \in \nu' \) and \( \tilde{m} \in M^k(\Pi, \nu; \epsilon) \). Clearly, when \( \epsilon' = \infty \) the desired subset relation holds, as \( M^k(\Pi', \nu'; \infty) = M \supseteq M^k(\Pi, \nu; \epsilon) \) for any choices of sets, orders and \( \epsilon \). Additionally, when \( M, \nu \) and \( \nu' \) are bounded, we know that \( \tilde{r}_{\max} \) and \( v_{\max} \) are finite. Thus, by selecting a finite \( \epsilon' \geq 2 \max\{\tilde{r}_{\max}, r_{\max}\} \frac{1-\gamma^K}{1-\gamma} + \gamma^K v_{\max} \), we obtain \( \tilde{m} \in M^k(\Pi', \nu'; \epsilon') \) and thus \( M^k(\Pi, \nu; \epsilon) \subseteq M^k(\Pi', \nu'; \epsilon') \) as needed.
Proposition 2. For any $\tilde{m} \in \mathcal{M}^k(\Pi, \mathcal{V}; \epsilon)$ it follows that
\[ \|v_{\pi_{\epsilon}} - v_*\| \leq 2 \cdot E_\epsilon(\Pi, \mathcal{V}, k \mid \Pi, \infty), \]
where $\pi_{\epsilon}$ is any optimal policy of $\tilde{m}$.

Proof. From Proposition [1] we know that a minimum tolerated error, $\epsilon' = E_\epsilon(\Pi, \mathcal{V}, k \mid \Pi, \infty)$, exists such that $\mathcal{M}^k(\Pi, \mathcal{V}; \epsilon') \subseteq \mathcal{M}^\infty(\Pi; \epsilon')$. We can then consider the performance of models in $\mathcal{M}^\infty(\Pi; \epsilon')$. For any $\tilde{m} \in \mathcal{M}^\infty(\Pi; \epsilon')$ we can write:
\[ 0 \geq \tilde{v}_{\pi_{\epsilon}}(s) - \tilde{v}_{\pi_{\epsilon}}(s) = (\tilde{v}_{\pi_{\epsilon}}(s) - v_{\pi_{\epsilon}}(s)) + (v_{\pi_{\epsilon}}(s) - v_{\pi_{\epsilon}}(s)) \]
for any $s \in \mathcal{S}$ where $\pi_{\epsilon}$ and $\tilde{\pi}_{\epsilon}$ are arbitrary optimal policies in the environment and $\tilde{m}$ respectively and $\tilde{v}_{\epsilon}$ denotes the model’s value of a policy $\pi$.

Since $\tilde{m} \in \mathcal{M}^\infty(\Pi; \epsilon')$ we know the first and third terms are bounded below by $-\epsilon'$, giving:
\[ 0 \geq v_{\pi}(s) - v_{\pi_{\epsilon}}(s) - 2\epsilon' \]
\[ \Rightarrow 2\epsilon' \geq v_{\pi}(s) - v_{\pi_{\epsilon}}(s) \geq 0 \]
\[ \Rightarrow \|v_{\pi} - v_{\pi_{\epsilon}}\| \leq 2\epsilon', \]
as needed. \(\square\)

Proposition 3. For any $\epsilon \in \mathbb{R}_+^+$, $\Pi \subseteq \Pi$, $\mathcal{V} \subseteq \mathcal{V}$ such that $v \in \mathcal{V} \implies T_{\pi}v \in \mathcal{V}$ $\forall \pi \in \Pi$ and $k, K \in \mathbb{Z}_+$ such that $k$ divides $K$, we have that
\[ \mathcal{M}^k(\Pi, \mathcal{V}; \epsilon) \subseteq \mathcal{M}^K(\Pi, \mathcal{V}; \epsilon(1-\gamma K)). \] (10)

Proof. Let $K = nk$, and consider a model $\tilde{m} \in \mathcal{M}^k(\Pi, \mathcal{V}; \epsilon)$. It follows for any $\pi \in \Pi$ and $v \in \mathcal{V}$ that
\[ \|\tilde{T}_\pi^Kv - T_{\pi}^Kv\| = \|\tilde{T}_\pi^{nk}T_{\pi}^{K-nk}v - T_{\pi}^{nk}T_{\pi}^{K-nk}v\| \]
\[ = \|\tilde{T}_\pi^{nk}T_{\pi}^{K-nk}v - T_{\pi}^{nk}T_{\pi}^{K-nk}v + \tilde{T}_\pi^{nk}T_{\pi}^{K-nk}v - \tilde{T}_\pi^{nk}T_{\pi}^{K-nk}v\| \]
\[ \leq \|\tilde{T}_\pi^{nk}T_{\pi}^{K-nk}v - T_{\pi}^{nk}T_{\pi}^{K-nk}v\| + \|\tilde{T}_\pi^{nk}T_{\pi}^{K-nk}v - \tilde{T}_\pi^{nk}T_{\pi}^{K-nk}v\| \]
\[ \leq \epsilon + \gamma^k\|\tilde{T}_\pi^{K-nk}v - T_{\pi}^{K-nk}v\| \]
\[ \leq \epsilon + \gamma^k\|\tilde{T}_\pi^{K-nk}v - T_{\pi}^{K-nk}v\| \]

where (1) follows from the assumption on $\mathcal{V}$ and (2) follows from the fact that $\tilde{T}_\pi$ is a contraction.

Next, using induction we can say that:
\[ \|\tilde{T}_\pi^{nk}v_{\pi} - T_{\pi}^{nk}v_{\pi}\| \leq \epsilon \left(1 + \gamma^k + \gamma^{2k} + \ldots + \gamma^{(n-1)k}\right) \]
\[ = \epsilon \cdot \sum_{t=0}^{n-1} \gamma^{kt} \]
\[ = \epsilon \cdot \frac{1-(\gamma^k)^n}{1-\gamma^k} \]
\[ = \epsilon \cdot \frac{1-\gamma^K}{1-\gamma^k} \]

where the last equality follows because $K = nk$.

This suffices to show that $\tilde{m} \in \mathcal{M}^K(\Pi, \mathcal{V}; \epsilon \cdot \frac{1-\gamma^K}{1-\gamma^k})$ and thus: $\mathcal{M}^k(\Pi, \mathcal{V}; \epsilon) \subseteq \mathcal{M}^K(\Pi, \mathcal{V}; \epsilon \cdot \frac{1-\gamma^k}{1-\gamma^k})$ as needed. \(\square\)

Corollary 1. For any set of policies $\Pi \subseteq \Pi$, set of functions $\mathcal{V} \subseteq \mathcal{V}$ such that $\{v_{\pi} : \pi \in \Pi\} \subseteq \mathcal{V}$ and $k, K \in \mathbb{Z}_+$, it follows that
\[ \mathcal{M}^k(\Pi, \mathcal{V}; \epsilon) \subseteq \mathcal{M}^\infty(\Pi; \frac{\epsilon}{1-\gamma^k}). \] (11)
Proof.

\[ M^k(\Pi, V; \epsilon) = \bigcap_{\pi \in \Pi} \bigcap_{v \in V} M^k(\{\pi\}, \{v\}; \epsilon) \subseteq \bigcap_{\pi \in \Pi} M^k(\{\pi\}, \{v_\pi\}; \epsilon) \]  

(23)

where the subset-relation holds from our assumption that \( \{ v_\pi : \pi \in \Pi \} \subseteq V \).

Next we examine \( \tilde{m} \in M^k(\{\pi\}, \{v_\pi\}; \epsilon) \) for individual \( \pi \in \Pi \). We know that for any such model:

\[ \| \tilde{T}_n^k v_\pi - v_\pi \| \leq \| \tilde{T}_n^k v_\pi - \tilde{T}_n^k v_\pi \| + \| \tilde{T}_n^k v_\pi - v_\pi \| \leq \gamma^k \| \tilde{T}_n^{(n-1)k} v_\pi - v_\pi \| + \epsilon. \]

By repeatedly applying this inequality we can obtain:

\[ \| \tilde{T}_n^k v_\pi - v_\pi \| \leq \sum_{t=0}^{n-1} \epsilon \cdot \gamma^{tk} = \epsilon \cdot \frac{1 - \gamma^n}{1 - \gamma}. \]

Next, from the continuity of \( \| \cdot \| \), we can take limits to obtain:

\[ \epsilon \cdot \frac{1}{1 - \gamma} \geq \lim_{n \to \infty} \| \tilde{T}_n^k v_\pi - v_\pi \| = \lim_{n \to \infty} \| \tilde{T}_n^k v_\pi - v_\pi \| = \| \tilde{v}_\pi - v_\pi \|, \]

giving us that \( M^k(\{\pi\}, \{v_\pi\}; \epsilon) \subseteq M^\infty(\{\pi\}; \epsilon \cdot \frac{1}{1 - \gamma}) \). We can plug this result back into Eq. (23) to obtain:

\[ M^k(\Pi, V; \epsilon) \subseteq \bigcap_{\pi \in \Pi} M^k(\{\pi\}, \{v_\pi\}; \epsilon) \subseteq \bigcap_{\pi \in \Pi} M^\infty(\{\pi\}; \epsilon \cdot \frac{1}{1 - \gamma}) = M^\infty(\Pi; \epsilon \cdot \frac{1}{1 - \gamma}), \]

as needed.

\[ \square \]

**Proposition 4.** For any set of policies \( \Pi \subseteq \mathbb{P} \), set of functions \( V \subseteq \mathbb{V} \), \( c > 1 \) and error \( \epsilon \in \mathbb{R}^+ \), we have

\[ M^k(\Pi, \text{c-vspan}(V); \epsilon) \subseteq M^k(\Pi, V; \epsilon) \subseteq M^k(\Pi, \text{c-vspan}(V); c \cdot \epsilon). \]  

(13)

**Proof.** Clearly, \( V \subseteq \text{c-vspan}(V) \) and thus \( M^k(\Pi, \text{c-vspan}(V); \epsilon) \subseteq M^k(\Pi, V; \epsilon) \). We now prove that \( M^k(\Pi, V; \epsilon) \subseteq M^k(\Pi, V; c \cdot \epsilon) \). We first consider any \( \tilde{m} \in M^k(\Pi, V; \epsilon) \) and \( v' \in \text{c-vspan}(V) \). Since \( v' \in \text{c-vspan}(V) \) we can write \( v' = \sum_{i=1}^{n} \alpha_i v_i \) where \( v_i \in V \) for each \( i \) and \( \sum_{i=1}^{n} |\alpha_i| \leq c. \)

From here we observe:

\[ \| \tilde{T}_n^k v' - \tilde{T}_n^k v' \| = \| \tilde{T}_n^k \left( \sum_{i=1}^{n} \alpha_i v_i \right) - \tilde{T}_n^k \left( \sum_{i=1}^{n} \alpha_i v_i \right) \| \]

\[ \leq \| \sum_{i=1}^{n} \alpha_i (\tilde{T}_n^k v_i - \tilde{T}_n^k v_i) \| \]

\[ \leq \sum_{i=1}^{n} |\alpha_i| \| \tilde{T}_n^k v_i - \tilde{T}_n^k v_i \| \]

\[ \leq \sum_{i=1}^{n} |\alpha_i| \epsilon \]

\[ \leq c \cdot \epsilon \]

which shows that \( M^k(\Pi, \text{c-vspan}(V); c \cdot \epsilon) \) as needed. \( \square \)

**Corollary 2.** When either \( c = 1 \) or \( \epsilon = 0 \), for any \( \Pi \subseteq \mathbb{P} \), \( V \subseteq \mathbb{V} \) it follows that

\[ M^k(\Pi, V; \epsilon) = M^k(\Pi, \text{c-vspan}(V); \epsilon). \]  

(14)

**Proof.** The proof follows directly from Proposition [4] When either \( c \in \{0, 1\} \) the left-most and right-most terms in Eq. (13) are equal, squeezing \( M^k(\Pi, V; \epsilon) = M^k(\Pi, \text{c-vspan}(V); \epsilon) \) as needed. \( \square \)

**Proposition 5.**
1. (Asymmetry) For any $\mathcal{V} \subseteq \mathcal{V}' \subseteq \mathcal{V}'' \subseteq \mathcal{V}$ it follows that

$$0 = \beta(\mathcal{V}||\mathcal{V}) \leq \beta(\mathcal{V}||\mathcal{V}') \leq \beta(\mathcal{V}||\mathcal{V}'')$$

and

$$0 = \beta(\mathcal{V}'||\mathcal{V}'') \leq \beta(\mathcal{V}'||\mathcal{V}') \leq \beta(\mathcal{V}||\mathcal{V}')$$.

2. (Convex, Compact $\mathcal{V}$) When $\mathcal{V}$ is convex and compact it follows that

$$\beta(\mathcal{V}||\mathcal{V}') = \beta(\mathcal{V}||\text{1-vspan}(\mathcal{V}')).$$

Proof.

1. Recall $\beta(\mathcal{V}||\mathcal{V}') = \max_{v' \in \mathcal{V}'} \min_{v \in \mathcal{V}} \|v' - v\|$. Increasing the size of $\mathcal{V}'$ means that more elements can be maximized over, thereby increasing $\beta(\mathcal{V}||\mathcal{V}')$. Similarly, increasing the size of $\mathcal{V}$ means that more elements can be minimized over, thereby decreasing $\beta(\mathcal{V}||\mathcal{V}')$. When $\mathcal{V} = \mathcal{V}'$, we know that

$$0 \leq \beta(\mathcal{V}||\mathcal{V}') = \max_{v' \in \mathcal{V}'} \min_{v \in \mathcal{V}} \|v' - v\| \leq \max_{v' \in \mathcal{V}'} \|v' - v''\| = 0,$$

where second inequality follows since $\mathcal{V} = \mathcal{V}'$.

2. We begin by considering the function $g(v') = \min_{v \in \mathcal{V}} \|v - v'\|$. We begin by showing that this function is convex. Consider $v_1', v_2' \in \mathcal{V}'$ and denote $v_1 = \arg \min_{v \in \mathcal{V}} \|v - v_1'\|$ and $v_2 = \arg \min_{v \in \mathcal{V}} \|v - v_2'\|$. Then for any $\lambda \in [0, 1]$ we can write:

$$\lambda g(v_1') + (1 - \lambda) g(v_2') = \lambda \|v_1' - v_1\| + (1 - \lambda) \|v_2' - v_2\|$$

$$\geq \|(\lambda v_1' + (1 - \lambda)v_2') - (\lambda v_1 + (1 - \lambda)v_2)\|$$

(25)

since $\mathcal{V}$ is convex $(\lambda v_1 + (1 - \lambda)v_2) \in \mathcal{V}$, thus:

$$\|(\lambda v_1' + (1 - \lambda)v_2') - (\lambda v_1 + (1 - \lambda)v_2)\| \geq \min_{v \in \mathcal{V}} \|(\lambda v_1' + (1 - \lambda)v_2') - v\|$$

$$= \lambda g(v_1') + (1 - \lambda) g(v_2')$$

(26)

which suffices to show that $g$ is a convex function.

Next we consider any element $v' \in \text{1-vspan}(\mathcal{V}')$ such that $v' = \sum_i \alpha_i v_i'$ with $\sum_i \alpha_i = 1$ and $\alpha_i \geq 0$ for all $i$. We can then write:

$$g(v') = g(\sum_i \alpha_i v_i') \leq \sum_i \alpha_i g(v_i') \leq \max_i g(v_i') \leq \max_{v' \in \mathcal{V}'\prime} g(v') = \beta(\mathcal{V}||\mathcal{V}')$$

(27)

Since $g(v') \leq \beta(\mathcal{V}||\mathcal{V}')$ for every $v' \in \text{1-vspan}(\mathcal{V}')$ it then follows that

$$\beta(\mathcal{V}||\text{1-vspan}(\mathcal{V}')) = \max_{v' \in \text{1-vspan}(\mathcal{V}')} g(v') \leq \beta(\mathcal{V}||\mathcal{V}')$$.

(28)

We obtain the reverse equality by noting that $\mathcal{V}' \subseteq \text{1-vspan}(\mathcal{V}')$ and thus $\beta(\mathcal{V}||\mathcal{V}') \leq \beta(\mathcal{V}||\text{1-vspan}(\mathcal{V}'))$. Hence $\beta(\mathcal{V}||\text{1-vspan}(\mathcal{V}')) = \beta(\mathcal{V}||\mathcal{V}')$ as needed.

$\blacksquare$

Proposition 6. For any $\Pi \in \Pi$, $\mathcal{V}, \mathcal{V}' \in \mathcal{V}$ and $\epsilon \in \mathbb{R}^+$, it follows that

$$\mathcal{M}^k(\Pi, \mathcal{V}; \epsilon) \subseteq \mathcal{M}^k(\Pi, \mathcal{V}'; \epsilon + 2\gamma^k \beta(\mathcal{V}||\mathcal{V}')),$$

moreover, if $\mathcal{V}$ is convex and compact, we obtain:

$$\mathcal{M}^k(\Pi, \mathcal{V}; \epsilon) \subseteq \mathcal{M}^k(\Pi, \text{1-vspan}(\mathcal{V}'); \epsilon + 2\gamma^k \beta(\mathcal{V}||\mathcal{V}')).$$

Proof. Fix an arbitrary model $\tilde{m} \in \mathcal{M}^k(\Pi, \mathcal{V}; \epsilon)$ and any $\pi \in \Pi$. We now select some $v' \in \mathcal{V}'$ and examine the tolerance with which $\tilde{m}$ is value equivalent with respect to $\{\pi\}$ and $\{v'\}$.
Notice that for any \( v \in \mathcal{V} \) we can write
\[
\| \hat{T}_v^k v' - T_v^k v' \| = \| \hat{T}_v^k v' - \hat{T}_v^k v + \hat{T}_v^k v - \hat{T}_v^k v \|
\leq \| \hat{T}_v^k v' - \hat{T}_v^k v \| + \| \hat{T}_v^k v - \hat{T}_v^k v' \|
= \| \hat{T}_v^k v' - \hat{T}_v^k v \| + \| \hat{T}_v^k v - \hat{T}_v^k v + \hat{T}_v^k v - \hat{T}_v^k v \|
\leq \| \hat{T}_v^k v' - \hat{T}_v^k v \| + \| \hat{T}_v^k v - \hat{T}_v^k v \| + \| \hat{T}_v^k v - \hat{T}_v^k v' \| \tag{29}
\]
where (1) follows from the Bellman operators \( \hat{T}_v \) and \( T_v \) being contractions and (2) follows the assumption that \( \tilde{m} \in \mathcal{M}^k(\Pi, \mathcal{V}) \).

Since the above upper bound on \( \| \hat{T}_v^k v' - T_v^k v' \| \) holds for any \( v \in \mathcal{V} \) we can write that
\[
\| \hat{T}_v^k v' - T_v^k v' \| \leq \epsilon + 2\gamma^k \min_{v' \in \mathcal{V}} \| v' - v \|. \tag{30}
\]
Thus far we have shown that \( \mathcal{M}^k(\Pi, \mathcal{V}; \epsilon) \subseteq \mathcal{M}^k(\Pi, \{ v' \}; \epsilon + 2\gamma^k \min_{v' \in \mathcal{V}} \| v' - v \|) \). To find a tolerance that holds for all \( v' \in \mathcal{V} \) we simply take a maximum over the element-wise tolerance:
\[
\max_{v' \in \mathcal{V}} \epsilon + 2\gamma^k \min_{v' \in \mathcal{V}} \| v' - v \| = \epsilon + 2\gamma^k \beta(\mathcal{V}||\mathcal{V}) \tag{31}
\]
This completes the proof.

**Theorem 2.** For any \( \tilde{m} \in \mathcal{M}^k(\Pi, \mathcal{V}; \epsilon) \) it follows that
\[
\| v_\pi - v_{\tilde{m}} \| \leq \frac{2}{1-\gamma^k} \min_{c \geq 1} (c \cdot \epsilon + 2\gamma^k \beta(c\text{-vspan}(\mathcal{V})||\mathcal{V})) , \tag{17}
\]
where \( \hat{\pi}_* \) is an optimal policy of \( \tilde{m} \).

**Proof.** From Theorem[1] we know by tolerating an error of
\[
\epsilon' = \frac{1}{1-\gamma^k} \min_{c \geq 1} (c \cdot \epsilon + 2\gamma^k \beta(\text{c-vspan}(\mathcal{V})||\mathcal{V})) ,
\]
that \( \mathcal{M}^k(\Pi, \mathcal{V}; \epsilon) \subseteq \mathcal{M}^\infty(\Pi; \epsilon') \). Thus \( \mathcal{E}_*(\Pi, \mathcal{V}, k || \Pi, \infty) \leq \epsilon' \). By applying Proposition[2] we obtain \( \| v_\pi - v_{\tilde{m}} \| \leq 2\epsilon' \) as needed.

**Corollary 3.** Let \( \hat{\mathcal{V}}_\Pi = \{ \hat{v}_\pi : \pi \in \Pi \} \) be a set of approximate value functions satisfying \( \| v_\pi - \hat{v}_\pi \| \leq \epsilon_{\text{approx}} \) for all \( \pi \in \Pi \). Then for any \( \tilde{m} \in \mathcal{M}^k(\Pi, \hat{\mathcal{V}}_\Pi; \epsilon) \) it follows that:
\[
\| v_\pi - v_{\tilde{m}} \| \leq \frac{2(\epsilon + 2\gamma^k \epsilon_{\text{approx}})}{1-\gamma^k} ,
\]
where \( \hat{\pi}_* \) is any optimal policy in \( \tilde{m} \).

**Proof.** From the definition of \( \hat{\mathcal{V}}_\Pi \), we know that \( \beta(\mathcal{V}_\Pi||\hat{\mathcal{V}}_\Pi) \leq \epsilon_{\text{approx}} \). Thus by Proposition[6] and Corollary[1] we know that
\[
\mathcal{M}^k(\Pi, \hat{\mathcal{V}}_\Pi; \epsilon) \subseteq \mathcal{M}^k(\Pi, \mathcal{V}_\Pi; \epsilon + 2\gamma^k \epsilon_{\text{approx}}) \subseteq \mathcal{M}^\infty(\Pi; \frac{\epsilon + 2\gamma^k \epsilon_{\text{approx}}}{1-\gamma^k}) ,
\]
thus \( \mathcal{E}_*(\Pi, \hat{\mathcal{V}}_\Pi, k || \Pi, \infty) \leq \frac{\epsilon + 2\gamma^k \epsilon_{\text{approx}}}{1-\gamma^k} \), which gives us the desired performance bound by an application of Theorem[2].

\[17\]