A Appendix

A.1 Statistical Analysis

Lemma 1. *Given I indices, and one index is chosen at each round equally randomly. The expected number of rounds of choosing all indices at least once is*

$$I\left(\frac{1}{I}+\frac{1}{I-1}+\cdots+\frac{1}{1}\right),$$

which is the same as

$$I\int_0^\infty \left(1-(1-e^{-t})^I\right)dt.$$

Proof. We denote the expected number of rounds to choose exactly i indices at least once as $E(i)$. Then we have $E(1)=1$, because, after the first round, one index is chosen. After the first round, the expected number of rounds to choose a new index is $\frac{I}{I-1}$, because one of the remaining $I-1$ out of the total I indices needs to be chosen. That is, $E(2)=E(1)+\frac{I}{I-1}$. Similarly, we have

$$E(i)=E(i-1)+\frac{I}{I+1-i}, \qquad \forall i=2,\ldots,I.$$

Thus, we have

$$E(I)=E(I-1)+I=E(I-2)+\frac{I}{2}+\frac{I}{1}=\cdots=I\left(\frac{1}{I}+\frac{1}{I-1}+\cdots+\frac{1}{1}\right).$$

The lemma is proved. □

It shows that the expected number of rounds to choose all indices at least once is $I\log(I)$ when $I\to\infty$. This proof can not be generalized to the case for choosing all indices at least m times for $m\geq 2$. Therefore, we provide alternative proof for it [34, Example 5.17].

Alternative proof of Lemma 1. This proof considers picking the indices as Poisson processes. Assume that the Poisson process to choose one index has a rate $\lambda=1$. Since the index is chosen equally randomly, choosing the jth index also follows a Poisson process with a rate $1/I$ for any j [34, Proposition 5.2]. We let X_j be the time to choose the first index j, and

$$X=\max_{1\leq j\leq I} X_j \tag{7}$$

is the time all indices are chosen at least once. Since all X_j are independent with rate $1/I$, we have

$$\begin{aligned}P\{X<t\}=&P\{\max_{1\leq j\leq I} X_j<t\}=P\{X_j<t, \text{ for } j=1,\ldots,I\}\\=&(1-e^{-t/I})^I.\end{aligned}$$

Therefore, we have

$$\mathbb{E}[X]=\int_0^\infty P\{x>t\}dt=\int_0^\infty \left(1-(1-e^{-t/I})^I\right)dt$$

We let N be the number of rounds to choose all indices at least one, and T_i be the ith interarrival time of the Poisson process for choosing one index. Then we have

$$X=\sum_{i=1}^N T_i,$$

and T_i are independent. Thus we have

$$\mathbb{E}[X|N]=N\mathbb{E}[T_i]=N,$$

and which gives

$$\mathbb{E}[X] = \mathbb{E}\{\mathbb{E}[X|N]\} = \mathbb{E}[N].$$

Thus we have

$$\mathbb{E}[N] = \int_0^\infty \left(1 - (1 - e^{-t/I})^I\right) dt = I \int_0^\infty \left(1 - (1 - e^{-t})^I\right) dt.$$

The lemma is proved. □

Next, we will present the lemma for choosing each index at least m times.

Lemma 2. *Given I indices, and one index is chosen at each round equally randomly. The expected number of rounds of choosing all indices at least m times is*

$$I \int_0^\infty \left(1 - (1 - S_m(t)e^{-t})^I\right) dt,$$

where

$$S_m(y) := 1 + y + \frac{y^2}{2!} + \cdots + \frac{y^{m-1}}{(m-1)!} = \sum_{l=0}^{m-1} \frac{y^l}{l!}. \tag{8}$$

Proof. We consider picking the indices as Poisson processes again. Assume that the Poisson process to choose one index has a rate $\lambda = 1$. Since the index is chosen equally randomly, choosing the jth index also follows a Poisson process with a rate of $1/I$ for any j. We let X_j be the time to choose index j for the mth time, and

$$X = \max_{1 \le j \le I} X_j \tag{9}$$

is the time all indices are chosen at least m times. Since all X_j are independent with rate $1/I$, we have

$$\begin{aligned} P\{X < t\} =& P\{\max_{1 \le j \le I} X_j < t\} = P\{X_j < t, \text{ for } j = 1, \dots, I\} \\ =& (1 - S_m(t/I)e^{-t/I})^I. \end{aligned}$$

Therefore, we have

$$\mathbb{E}[X] = \int_0^\infty P\{x > t\} dt.$$

We let N be the number of rounds to choose all indices at least m times, and T_i be the ith interarrival time of the Poisson process for choosing one index. Then we have

$$X = \sum_{i=1}^N T_i,$$

and T_i are independent. Thus we have

$$\mathbb{E}[X|N] = N\mathbb{E}[T_i] = N,$$

and which gives

$$\mathbb{E}[X] = \mathbb{E}\{\mathbb{E}[X|N]\} = \mathbb{E}[N].$$

Thus we have

$$\mathbb{E}[N] = \int_0^\infty \left(1 - (1 - S_m(t/I)e^{-t/I})^I\right) dt = I \int_0^\infty \left(1 - (1 - S_m(t)e^{-t})^I\right) dt.$$

The lemma is proved. □

It shows that the expected number of rounds to choose all indices at least once is $I \log(I) + I(m - 1) \log \log I$ when $I \to \infty$ [35].

A.2 Formal Definition of Selective Aggregation Scheme

Formally speaking, let $\mathcal{M} \subset \mathcal{N}$ be the set of selected clients from the client pool from which the server pulls model parameters at round j. Let $\theta_{[i,k]}$ be the k^{th} parameter of layer i of the global model and $\theta_{m,[i,k]}$ be the k^{th} parameter of layer i of client m. We denote $\mathcal{M}_k \subset \mathcal{M}$ as the set of clients updating the k^{th} parameter. The model parameters are aggregated as follows:

$$\theta_{[i,k]} = \frac{1}{\sum_{m \in \mathcal{M}_k} p_m} \sum_{m \in \mathcal{M}_k} p_m \theta_{m,[i,k]}, \tag{10}$$

The client weight p_m is assigned based on factors like the client model capacity, the number of data points a client has, etc. Throughout the paper, unless otherwise stated, the weight of all clients is assumed to be the same, i.e, $p_m = 1/N$.

A.3 Ablation Study: Impact of Different Weighing Schemes

[12] reported that weighting clients is important to improving model accuracy. Therefore, we did an ablation study and evaluated three client weighting schemes: (1) **model size-based weighting scheme**: client weight is proportional to the number of kernels in the model; (2) **model update-based weighting scheme**: client weight is proportional to the number of updates; and (3) **hybrid weighting scheme**: client weight is proportional to both (1) model size and (2) model update.

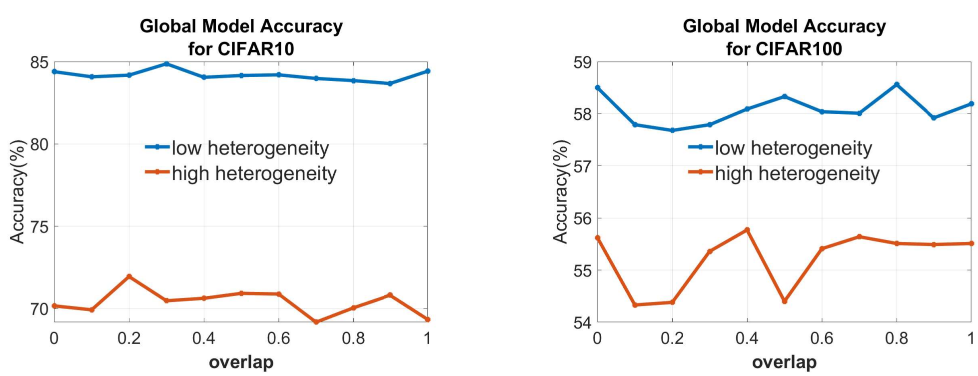
Table 5 lists the results. As shown, the performance of the three weighting schemes is not significantly better than the non-weighting scheme. Therefore, we used the non-weighting scheme in `FedRolex`.

Table 5: Impact of weighting schemes on model accuracy under high data heterogeneity.

	Weighting Scheme	Local Model Accuracy	Global Model Accuracy
CIFAR-10	Non-Weighting	95.95 (±0.81)	**69.44 (±1.50)**
	Model Size-based	95.98 (±0.67)	69.09 (±1.42)
	Model Update-based	96.01 (±0.71)	68.83 (±0.89)
	Hybrid	**96.05 (±0.96)**	68.78 (±0.89)
CIFAR-100	Non-Weighting	**81.58 (±0.59)**	56.57 (±0.15)
	Model Size-based	81.23 (±1.56)	**56.99 (±0.27)**
	Model Update-based	81.23 (±1.07)	56.63 (±0.36)
	Hybrid	81.49 (±1.07)	56.71 (±0.20)

A.4 Ablation Study: Impact of Overlapping Kernels

We also studied the impact of overlapping kernels between rounds using ResNet-18 and CIFAR-10/CIFAR-100 as an example. Specifically, we extracted sub-models using a rolling window that



(i) (ii)

Figure 6: Impact of inter-round kernel overlap on global model accuracy under low and high data heterogeneity for (i) CIFAR-10 and (ii) CIFAR-100.

advances and loops over all the kernels of each convolution layer in the global model in strides. Let the degree of overlap between each stride of the rolling window be $r \in [0, 1]$. In each iteration, each convolution layer in the global model is advanced by $1 + \lfloor \beta_n (1 - r) K_i \rfloor$ where $\lfloor \cdot \rfloor$ is the floor function. In `FedRolex`, $r = 1$, i.e., the kernels are advanced by 1 from one iteration to the next iteration.

Figure 6 shows the impact of different r on global model accuracy. As shown, the value of r does have some influence on the global model accuracy, but the impact is non-linear and inconsistent.

A.5 Ablation Study: Impact of Client Participation Rate

In our main paper, we followed prior arts [14, 24, 15, 36, 9] and used a 10% client participation rate. To examine the effect of client participation rate, we conducted experiments with both lower (5%) and higher (20%) client participation rates using CIFAR-10 as an example for `FedRolex`, HeteroFL and Federated Dropout.

The results are summarized in Table 6. As shown, `FedRolex` consistently outperforms both Federated Dropout and HeteroFL across 5%, 10% and 20% client participation rates.

Table 6: Performance of FedRolex, HeteroFL, and Federated Dropout under different client participation rates.

		Client Participation Rate		
		5%	10%	20%
CIFAR-10	HeteroFL	48.43 (+/- 1.78)	63.90 (+/-2.74)	65.07 (+/- 2.17)
	Federated Dropout	42.06 (+/- 1.29)	46.64 (+/-3.05)	55.20 (+/- 4.64)
	`FedRolex`	**57.90 (+/- 2.72)**	**69.44 (+/-1.50)**	**71.85 (+/- 1.22)**

A.6 Communication and Computation Costs of FedRolex

To calculate the communication cost, we use the average size of the models sent by all the participating clients per round as the metric. To calculate the computation overhead, we calculate the FLOPs and numbers of parameters in the models of all the participating clients per round and take the average as the metric. To put these metrics in context, we also calculate the upper and lower bounds of the communication cost and computation overhead (i.e., all the clients were using the same largest model and smallest model, respectively).

Table 7 lists the results. As shown, compared to the upper bound, `FedRolex` significantly reduces the communication cost and computation overhead while being able to achieve comparable model accuracy. Compared to the lower bound, although `FedRolex` has higher communication cost and computation overhead, the model accuracy achieved is much higher than the lower bound. These results indicate that `FedRolex` is able to achieve comparable high model accuracy as the upper bound with much less communication cost and computation overhead.

Table 7: Computation and communication costs of FedRolex compared to upper and lower bounds represented by homogeneous settings with largest and smallest models respectively.

	Homogeneous (largest)	**FedRolex**	**Homogeneous (smallest)**
Average Number of Parameters per Client (Million)	11.1722	2.9781232	0.04451
Average FLOPs per Client (Million)	557.656	149.048384	2.41318
Average Model Size per Client (MB)	42.62	11.36	0.17

A.7 Experimental Setup Details

Experimental Setup Details for Table 3. The experimental setup for PT-based methods is listed in Table 8. The experimental setup for model-homogeneous baselines was slightly different from the PT-based methods and hence is listed separately in Table 9.

Experimental Setup Details for Figure 3. The experimental setup details are tabulated in Tables 10 and 11.

Experimental Setup Details for Figure 4. The experimental setup details are tabulated in Table 12.

Table 8: Experimental setup details of PT-based methods in Table 3 on CIFAR-10, CIFAR-100 and Stack Overflow.

		CIFAR-10	CIFAR-100	Stack Overflow
Local Epoch		1	1	1
Cohort SIze		10	10	200
Batch Size		10	24	24
Initial Learning Rate		2.00E-04	1.00E-04	2.00E-04
Decay Schedule	High Data Heterogeneity	800, 1500	1000, 1500	600, 800
	Low Data Heterogeneity	800, 1250	1000, 1500	
Decay Factor		0.1	0.1	0.1
Communication Rounds	High Data Heterogeneity	2500	3500	1200
	Low Data Heterogeneity	2000	3500	
Optimizer		SGD	SGD	SGD
Momentum		0.9	0.9	0.9
Weight Decay		5.00E-04	5.00E-04	5.00E-04

Table 9: Experimental setup details of model-homogeneous baselines in Table 3 on CIFAR-10 and CIFAR-100 and Stack Overflow.

		CIFAR-10	CIFAR-100	Stack Overflow
Local Epoch		1	1	1
Cohort Size		10	10	200
Batch Size		10	24	24
Initial Learning Rate		2.00E-04	1.00E-04	2.00E-04
Decay Schedule	High Data Heterogeneity	500, 1000	1000, 1500	300
	Low Data Heterogeneity	500, 1000	1000, 1500	
Decay Factor		0.1	0.1	0.1
Communication Rounds$\sim$	High Data Heterogeneity	1250	3500	1000
	Low Data Heterogeneity	1500	3500	
Optimizer		SGD	SGD	SGD
Momentum		0.9	0.9	0.9
Weight Decay		5.00E-04	5.00E-04	5.00E-04

Table 10: Experimental setup for results shown in Figure 3. ρ between 0.0 and 0.5 in 0.1 increments.

Dataset		ρ	0.0	0.1	0.2	0.3	0.4
CIFAR-10	High Heterogeneity	Decay Schedule	500, 1000	500, 1000	500, 1000	700, 1200	700, 1200
		Communication Rounds	1250	1250	1250	1500	1500
	Low Heterogeneity	Decay Schedule	500, 1000	500, 1000	500, 1000	700, 1200	700, 1200
		Communication Rounds	1250	1250	1250	1500	1500
CIFAR-100	High Heterogeneity	Decay Schedule	1000, 1500	1000, 1500	1000, 1500	1000, 1500	1000, 1500
		Communication Rounds	2000	2000	2000	2000	2000
	Low Heterogeneity	Decay Schedule	1000, 1500	1000, 1500	1000, 1500	1000, 1500	1000, 1500
		Communication Rounds	2000	2000	2000	2000	2000
Stack Overflow	High Heterogeneity	Decay Schedule	800	800	800	800	800
		Communication Rounds	1500	1500	1500	1500	1500
	Low Heterogeneity	Decay Schedule	800	800	800	800	800
		Communication Rounds	1500	1500	1500	1500	1500

A.8 Algorithm Pseudocodes

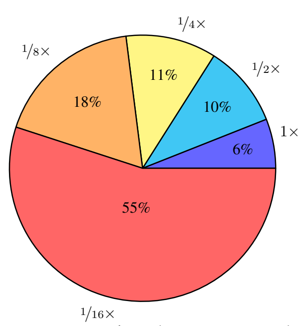
The pseudocodes for HeteroFL and Federated Dropout are given in Algorithms 2 and 3 respectively. Their differences from `FedRolex` are marked using blue color.

Table 11: Experimental setup for results shown in Figure 3. ρ between 0.5 and 1.0 in 0.1 increments.

Dataset		ρ	0.6	0.7	0.8	0.9	1.0
CIFAR-10	High Heterogeneity	Decay Schedule	700, 1200	700, 1200	500, 1000	500, 1000	500, 1000
		Communication Rounds	1500	1500	1250	1250	1250
	Low Heterogeneity	Decay Schedule	700, 1200	700, 1200	500, 1000	500, 1000	500, 1000
		Communication Rounds	1500	1500	1250	1250	1250
CIFAR-100	High Heterogeneity	Decay Schedule	1000, 1500	1000, 1500	1000, 1500	1000, 1500	1000, 1500
		Communication Rounds	2000	2000	2000	2000	2000
	Low Heterogeneity	Decay Schedule	1000, 1500	1000, 1500	1000, 1500	1000, 1500	1000, 1500
		Communication Rounds	2000	2000	2000	2000	2000
Stack Overflow	High Heterogeneity	Decay Schedule	800	800	800	800	800
		Communication Rounds	1500	1500	1500	1500	1500
	Low Heterogeneity	Decay Schedule	800	800	800	800	800
		Communication Rounds	1500	1500	1500	1500	1500

Table 12: Experimental setup for results shown in Figure 4

Dataset		γ	2	4	8	16
CIFAR-10	High Heterogeneity	Decay Schedule	800, 1200	800, 1200	800, 1200	800, 1200
		Communication Rounds	1500	1500	1500	1500
	Low Heterogeneity	Decay Schedule	800, 1200	800, 1200	800, 1200	800, 1200
		Communication Rounds	1500	1500	1500	1500
CIFAR-100	High Heterogeneity	Decay Schedule	800, 1200	800, 1200	800, 1200	800, 1200
		Communication Rounds	1500	1500	1500	1500
	Low Heterogeneity	Decay Schedule	800, 1200	800, 1200	800, 1200	800, 1200
		Communication Rounds	1500	1500	1500	1500
Stack Overflow	High Heterogeneity	Decay Schedule	800	800	800	800
		Communication Rounds	1500	1500	1500	1500
	Low Heterogeneity	Decay Schedule	800	800	800	800
		Communication Rounds	1500	1500	1500	1500



Model Capacity	Annual Household Income
$^1/_{16}\times$	$< \$75,000$
$^1/_{8}\times$	$\$75,000 - \$100,000$
$^1/_{4}\times$	$\$100,000 - \$150,000$
$^1/_{2}\times$	$\$150,000 - \$200,000$
$1\times$	$> \$200,000$

Figure 7: Mapping between real-world annual household income and model capacity.

Table 13: Experimental setup for Table 4 for CIFAR-10, CIFAR-100 and Stack Overflow.

		CIFAR-10	CIFAR-100	Stack Overflow
Local Epoch		1	1	1
Cohort SIze		10	10	200
Batch Size		10	24	24
Initial Learning Rate		2.00E-04	1.00E-04	2.00E-04
Decay Schedule	High Heterogeneity	800, 1500	1000, 1500	600, 800
	Low Heterogeneity	800, 1250	1000, 1500	
Decay Factor		0.1	0.1	0.1
Communication Rounds	High Heterogeneity	2500	3500	1200
	Low Heterogeneity	2000	3500	
Optimizer		SGD	SGD	SGD
Momentum		0.9	0.9	0.9
Weight Decay		5.00E-04	5.00E-04	5.00E-04

Algorithm 2: HeteroFL

Initialization ; $\theta^{(0)}, \mathcal{N}$
Input : $D_n\ \beta_n\ \forall n \in \mathcal{N}$,
Output : θ^J
Server Executes
for $j \leftarrow 0$ **to** $J-1$ **do**
  Sample subset $\mathcal{M}$ from $\mathcal{N}$
  Broadcast $\theta^{(j)}_{m,[i\ ;\ 0,1,\ \dots\ \lfloor \beta_n K_i \rfloor -1]} \forall i$ and $m \in \mathcal{M}$
  for *each* *client* $m \in \mathcal{M}$ **do**
    `clientStep(`$\theta^{(j)}_m$, D_m`)`
  end
  Aggregate $\theta^{(j+1)}_{[i,k]}$ according to Equation (10)
end
Subroutine `clientStep(`$\theta^{(j)}_n$, D_n`)`
  $m_n \longleftarrow len(D_n)$
  for $k \leftarrow 0$ **to** m_n **do**
    $\theta_n \longleftarrow \theta_n - \eta \nabla l(\theta_n; d_{n,k})$
  end
  return θ_n

Algorithm 3: Federated Dropout

Initialization ; $\theta^{(0)}, \mathcal{N}$
Input : $D_n\ \beta_n\ \forall n \in \mathcal{N}$,
Output : θ^J
Server Executes
for $j \leftarrow 0$ **to** $J-1$ **do**
  Sample subset $\mathcal{M}$ from $\mathcal{N}$
  Broadcast $\theta^{(j)}_{m,[i\ ;\ k_1,\dots,k_{\lfloor \beta_n K_i \rfloor}]} \forall i$ and $m \in \mathcal{M}$
  for *each* *client* $m \in \mathcal{M}$ **do**
    `clientStep(`$\theta^{(j)}_m$, D_m`)`
  end
  Aggregate $\theta^{(j+1)}_{[i,k]}$ according to Equation (10)
end
Subroutine `clientStep(`$\theta^{(j)}_n$, D_n`)`
  $m_n \longleftarrow len(D_n)$
  for $k \leftarrow 0$ **to** m_n **do**
    $\theta_n \longleftarrow \theta_n - \eta \nabla l(\theta_n; d_{n,k})$
  end
  return θ_n