Bridging the Gap from Asymmetry Tricks to Decorrelation Principles in Non-contrastive Self-supervised Learning

A Proof of Lemma 3.2

Proof of ***Lemma*** *3.2.* We can expand the loss (4), i.e.,

$$\mathcal{L} = \frac{1}{2}\mathbb{E}_x[\|W_\mathrm{p} f^1 - \mathrm{StopGrad}(f^2)\|^2]$$

as follows:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\mathbb{E}_x[\mathrm{tr}(W_\mathrm{p} f^1 f^{1\top} W_\mathrm{p}^\top) - 2\mathrm{tr}(\mathrm{StopGrad}(f^2) f^{1\top} W_\mathrm{p}^\top) + \mathrm{tr}(\mathrm{StopGrad}(f^2 f^{2\top}))] \\ &= \frac{1}{2}(\mathrm{tr}(W_\mathrm{p}\mathbb{E}_x[f^1 f^{1\top}]W_\mathrm{p}^\top) - 2\mathrm{tr}(\mathbb{E}_x[\mathrm{StopGrad}(f^2) f^{1\top}]W_\mathrm{p}^\top) + \mathrm{tr}(\mathbb{E}_x[\mathrm{StopGrad}(f^2 f^{2\top})])) \\ &= \frac{1}{2}(\mathrm{tr}(W_\mathrm{p} F^1 F^{1\top} W_\mathrm{p}^\top) - 2\mathrm{tr}(\mathrm{StopGrad}(F^2) F^{1\top} W_\mathrm{p}^\top) + \mathrm{tr}(\mathrm{StopGrad}(F^2 F^{2\top})])). \qquad (20)\end{aligned}$$

We used $\mathbb{E}_x[f^1 f^{1\top}] = F^1 F^{1\top}$, $\mathbb{E}_x[f^2 f^{1\top}] = F^2 F^{1\top}$, and $\mathbb{E}_x[f^2 f^{2\top}] = F^2 F^{2\top}$.

Taking derivatives of $\mathcal{L}$ with respect to W_p yields (7) since

$$\frac{\partial \mathcal{L}}{\partial W_\mathrm{p}} = \frac{1}{2}(2W_\mathrm{p} F^1 F^{1\top}) - F^2 F^{1\top} = W_\mathrm{p} F^1 F^{1\top} - F^2 F^{1\top}. \qquad (21)$$

Note that there is no gradient at $\mathrm{StopGrad}(\cdot)$. Similarly, the derivative with respect to $F^{1\prime}$ is given by (8) since

$$\frac{\partial \mathcal{L}}{\partial F^1} = \frac{1}{2}(2W_\mathrm{p}^\top W_\mathrm{p} F^1) - W_\mathrm{p}^\top F^2 = W_\mathrm{p}^\top W_\mathrm{p} F^1 - W_\mathrm{p}^\top F^2. \qquad (22)$$

□

B Proof of Lemma 3.3

Proof of ***Lemma*** *3.3.* Assume the mapping from the input $x \in \mathbb{R}^P$ to $f \in \mathbb{R}^D$ is given by a linear transformation $f = Wx$. (Or maybe affine $f = Wx + b$.) We will use a 'vectorized' representation of $W = [w_1, \ldots, w_D]^\top$ as $w = [w_1^\top, \ldots, w_D^\top]^\top \in \mathbb{R}^{DP}$. Now, suppose we change w as $w \to w + \delta w$, where we choose the gradient of L for δw as

$$\delta w = -\frac{\partial L}{\partial w}. \qquad (23)$$

Then, f will change as $f \to f + \delta f$ accordingly, where

$$\delta f = \frac{\partial f}{\partial w}\delta w = -\frac{\partial f}{\partial w}\frac{\partial L}{\partial w}. \qquad (24)$$

Since w affects the loss L only through f, we use chain rule to get

$$\frac{\partial L}{\partial w} = \left(\frac{\partial f}{\partial w}\right)^\top \frac{\partial L}{\partial f}. \qquad (25)$$

Submitted to 36th Conference on Neural Information Processing Systems (NeurIPS 2022). Do not distribute.

Thus, substituting this into (24) yields

$$\delta f = -\frac{\partial f}{\partial w}\left(\frac{\partial f}{\partial w}\right)^{\top}\frac{\partial L}{\partial f}. \tag{26}$$

Since $f = [w_1^{\top}x, \dots, w_D^{\top}x]^{\top}$,

$$\frac{\partial f}{\partial w} = \begin{bmatrix} x^{\top} & & & \\ & x^{\top} & & \\ & & \ddots & \\ & & & x^{\top} \end{bmatrix} (\in \mathbb{R}^{D\times DP}) \tag{27}$$

Thus,

$$\frac{\partial f}{\partial w}\left(\frac{\partial f}{\partial w}\right)^{\top} = \begin{bmatrix} x^{\top}x & & & \\ & x^{\top}x & & \\ & & \ddots & \\ & & & x^{\top}x \end{bmatrix} = (x^{\top}x)I \tag{28}$$

Substituting this into (26) yields

$$\delta f = -\frac{\partial f}{\partial w}\left(\frac{\partial f}{\partial w}\right)^{\top}\frac{\partial L}{\partial f} = -(x^{\top}x)\frac{\partial L}{\partial f}. \tag{29}$$

This states that when we move w in the direction δw of minimizing L, f moves in the direction of $-\partial L/\partial f$.

Next, we consider the effect of weight decay on w. Then δw becomes $-\partial L/\partial w - \eta w$. We consider the change $\delta f'$ of f due to $-\eta w$. This is given by

$$\delta f' = \frac{\partial f}{\partial w}(-\eta w) \tag{30}$$

Using (27),

$$\delta f' = -\eta \begin{bmatrix} x^{\top}w_1 \\ x^{\top}w_2 \\ \vdots \\ x^{\top}w_D \end{bmatrix} = -\eta W x = -\eta f \tag{31}$$

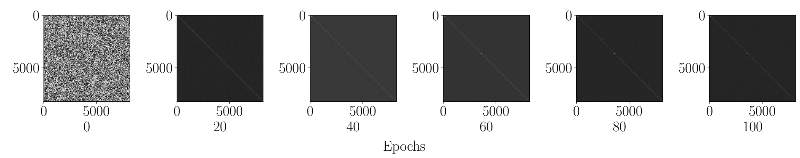
In conclusion, when we move w as $w \to w - \partial L/\partial w - \eta w$, f's change δf will be given by

$$\delta f = -(x^{\top}x)\frac{\partial L}{\partial f} - \eta f. \tag{32}$$

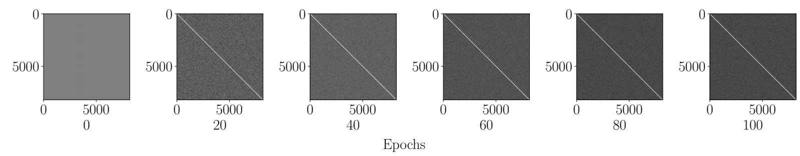
Supposing x to be an ImageNet image, we may think $x^{\top}x \sim \mathrm{const}$. Assuming $x^{\top}x = 1$, the above leads to $\dot{F} = -\partial L/\partial F - \eta F$, which is (11). $\square$

C Behavior of W_{p} in Ours

We also examine how W_{p} changes in the optimization of our proposed method. Following the same experimental setting as Sec. 5, we train a linear predictor with $W_{\mathrm{p}} \in \mathbb{R}^{8192\times 8192}$. As shown in Fig. 3, W_p approaches to a diagonal matrix during training process.



(a)



(b)

Figure 3: (a) W_p in our proposed method at different epochs of training time. (b) Normalized W_p for better visualization; each row of W_p is divided by its diagonal entry.

D More Results with a Broader Range of Configuration

Table 5 shows more results of our method with different configurations and hyperparameter settings.

Table 5: Results of our method on more configurations.

PROJECTOR	PREDICTOR	LR	#PARTITION	INV COEF	COV COEF	ACC@1
4096-256	256	0.3	1	1	1	54.7
4096-256	256	0.3	1	10	1	56.8
4096-256	256	0.3	1	1	10	53.1
4096-256	256	0.3	8	10	1	63.8
4096-256	256	0.3	32	1	1	54.8
4096-256	256	0.3	32	10	1	63.7
2048-2048-2048	2048	0.3	1	1	0	50.9
2048-2048-2048	2048	0.3	1	1	1	61.4
2048-2048-2048	2048	0.3	1	1	25	60.6
2048-2048-2048	2048	0.3	1	25	1	60.1
2048-2048-2048	2048	0.3	1	25	25	63.4
2048-2048-2048	2048	0.3	8	25	1	67.3
2048-2048-2048	2048	0.3	8	25	25	66.2
2048-2048-2048	2048	0.3	32	10	1	66.6
2048-2048-2048	2048	0.3	32	25	1	67.3
2048-2048-2048	2048	0.3	32	50	1	67.2
2048-2048-2048	2048	0.45	1	1	1	63.9
2048-2048-2048	2048	0.45	1	25	25	64.7
2048-2048-2048	2048	0.45	32	10	1	64.3
8192-8192-8192	8192	0.3	8	25	8	69.0

E Standardization layer

Feature standardization is computed as following, independent to partitions:

$$Std(f_i) = \frac{f_i - \mu_{i,sync}}{\sigma_{i,sync}} \tag{33}$$

,where f_i is the ith entry of f, $\mu_{i,sync}$ and $\sigma_{i,sync}$ are synchronized ith entry of mean and deviation respectively among devices.

F Pseudo-code of our proposed method

Algorithm 1 Our Proposed Method, PyTorch-like

```
# h: backbone + projector
# w: weight of predictor
# D: projector output size
# C_in, C_cov: coefficients
#
# In this pseudo-code, we assume number of partitions equals to number of gpus, and the following code is processed on a single gpu.
for x in loader:
    x1, x2 = aug1(x), aug2(x)
    f1, f2 = h(x1), h(x2)
    f1, f2 = std(f1), std(f2)

    # Processed by predictor
    p1, p2 = f1 @ w.T, f2 @ w.T
    inv_loss = (p1-f2).pow(2).mean() + (p2-f1).pow(2).mean()

    # Note: we do not collect cov from
    # different gpus
    wtw = w.T @ w
    n = p1.size(0) # Batch size per gpu
    cov1 = p1.T @ p1 / n
    cov2 = p2.T @ p2 / n
    cov_loss = (cov1-wtw).pow(2).sum() + (cov2-wtw).pow(2).sum()

    loss = C_in * inv_loss + C_cov / D * cov_loss
    loss.backward()
    update(f,w)


def std(f): # Standarization
    return SyncBN(affine=False)(f)
```