Reviewer #3

Thank you for your careful reading and very positive comments. We feel very encouraged.

Reviewer #4

Thank you for your careful reading and very constructive comments.

Q: *Proof techniques are mainly based on [4] (Bousquet et al. 2019) which limits the novelty.*

A: Thank you for your valuable review. We would like to emphasize the novelty of our work. As you point out, one of the key tools we make use of is a concentration inequality from [4], which considers a summation of n functions of n independent random variables. However, this concentration inequality does not fit the structure of pairwise learning. To address this discrepancy, we introduce novel and tricky decompositions of U-statistics (cf. Lines 33-34 and Lines 38-39 in the Appendix). Furthermore, the existing stability analysis (presented in [4]) requires the assumption of a bounded loss: for RRM (Regularized Risk Minimization), the loss needs to be bounded by $O(1/\sqrt{\sigma})$ (Lines 185-187). Thus, even in the case of *pointwise learning*, the stability analysis in [4] only yields loose bounds of rate $O((n\sigma)^{-1/2})$ when one takes into account the magnitude of the loss function—in contrast, we prove $O((n\sigma)^{-1} + n^{-1/2})$ in that case (cf. eq. (4.4)). This boundedness assumption can not be addressed by adding a constraint, as doing so would negate the main advantage of the RRM approach. We relax this boundedness assumption to a variance assumption on $\ell(\mathbf{w}^*; Z, \tilde{Z})$. Note that the expectation of $\ell(\mathbf{w}^*; Z, \tilde{Z})$ is $R(\mathbf{w}^*)$, which is small according to the definition of $\mathbf{w}^*$. Therefore, it is reasonable to assume that the variance of $\ell(\mathbf{w}^*; Z, \tilde{Z})$ is bounded. To achieve this relaxation, we use a novel application of Theorem 1 to $\tilde{\ell}(\mathbf{w}; z, \tilde{z}) = \ell(\mathbf{w}; z, \tilde{z}) - \ell(\mathbf{w}^*; z, \tilde{z})$ instead of $\ell(\mathbf{w}; z, \tilde{z})$ (cf. Line 107 in the Appendix). Moreover, we introduce a novel lemma (Lemma 2) to show $\big|\mathbb{E}_S\big[\tilde{\ell}(A(S); z, \tilde{z})\big]\big| = O(1/(\sqrt{n}\sigma))$ (cf. Line 105 in the Appendix) and apply Bernstein's inequality to address the concentration behavior of $\ell(\mathbf{w}^*; Z, \tilde{Z})$. We believe the techniques we develop, both to adapt to the structure of pairwise learning and to relax the boundedness assumption, are novel.

Other than this contribution, we would like to emphasize that we derive the first generalization bounds for SGD in pairwise learning, a setting where the existing bounds could not obtain non-trivial guarantees. We also introduce on-average stability for pairwise learning and use it to develop optimistic bounds in a low noise setting.

Q: *The claim on minimax optimality is not correct since the conditions do not match in upper and lower bounds.*

A: Thank you for your insightful comment. We promise to tone down our original informal formulation of the claim to reflect the facts about the conditions. We would like to mention that we also obtain the bound $O(n^{-\frac{1}{2}}\log n)$ for SGD, which does not require the objective function to be strongly convex. Thus, our bound for SGD is minimax optimal up to a logarithmic factor. We will study lower bounds of pairwise learning in the strongly convex setting in our future study.

Q: *The condition $\|\mathbf{w}_R^*\| = O(1)$ may be too strict.*

A: Thank you for indicating this. We use this condition to simplify the presentation of eq. (4.9). If we do not impose this condition, the upper bound in (4.9) becomes $O\big(\sqrt{R(\mathbf{w}_R^*)}\|\mathbf{w}_R^*\|n^{-\frac{1}{2}} + \|\mathbf{w}_R^*\|^2 n^{-1}\big)$ (cf. Line 227 in the Appendix).

Q: *They don't require the condition of bounded loss function, but for most of results the loss should be strongly convex.*

A: Thank you. To be precise, we require the strong convexity of F_S (loss function + regularizer), not that of loss functions. Previous work assumed loss functions to be bounded by a universal constant, as well as the strong convexity of F_S. However, strong convexity only guarantees that loss functions are bounded by $O(\sqrt{1/\sigma})$ (Line 185-187). Thus without a boundedness assumption, the existing stability analysis implies a loose bound even for *pointwise learning* [4].

Q: *Meaning of RRM in Line 130. λ should be σ in Line 197.* **A**: Thank you. RRM stands for "regularized risk minimization". We explain this abbreviation in Line 44. We will replace λ by σ in Line 197.

Reviewer #6

Thank you for your careful reading and very positive comments. We feel very encouraged.

Q: *The second paragraph of Section 2 exaggerates the merits of algorithmic stability a little bit.*

A: We agree with this point. We will further discuss the merit of the uniform convergence approach over the stability approach in providing very valuable information in the non-convex setting.

Q: *Reference [7] (Bousquet et al. 2019) could have been acknowledged a bit more.*

A: We agree and promise to acknowledge this important reference more.

Q: *Some Typos.* **A**: Thank you for your careful reading. We completely agree and will fix them in the revision.