- We greatly appreciate the reviewers for taking a close look at the paper and the proofs, and giving a detailed feedback.
- 2 We first address the common concerns raised.
- 3 Dependence on d in Lemma 1: While our focus has been on the dependence of our algorithm on the number of
- 4 plug-in calls, we understand why the reviewers would like the dependence on d to be made explicit. Below, we expand
- 5 the statistical error term in Lemma 1 to show the dependence on d, and will include this in the paper along with the
- 6 complete proof. This is the *same* dependence that the previous method of Narasimhan (2018) incurs [26].
- 7 For a n-class problem, let $\hat{g}^{\mathbf{a}}$, $\tilde{\mathbf{u}}^{\mathbf{a}} = \text{plug-in}(\mathbf{a})$ as in Algorithm 1. Then with probability $\geq 1 \delta$ over draw of N
- 8 examples from the data distribution, we have for all $\mathbf{a} \in \mathbb{R}^d$:

$$||C[\widehat{g}^{\mathbf{a}}] - \widetilde{\mathbf{u}}^{\mathbf{a}}||_2 = O\left(d\sqrt{\frac{d\log(d) + \log(Nn^2) + \log(1/\delta)}{N}}\right)$$

- 9 where the notation *O* only hides absolute constants. The proof follows from a straightforward application of a result 10 from Cesa-Bianchi & Haussler (1998) to bound the growth function.
- Proof of Proposition 2: Proposition 2 is straightforward and simply follows from expanding $\langle \mathbf{a}, C[h] \rangle$ as $\mathbf{E}_X \left[\sum_{y=1}^n \eta_y(X) \sum_{i=1}^d a_i \sigma_i(X,y,h(X)) \right]$. Hence the Bayes-optimal classifier h predicts for any given x, a label \widehat{y} that minimizes the inner term $\sum_{y=1}^n \eta_y(x) \sum_{i=1}^d a_i \sigma_i(x,y,\widehat{y})$, i.e. $h(x) = \operatorname{argmin}_{\widehat{y} \in [n]} \sum_{y=1}^n \eta_y(x) L_{y,\widehat{y}}(x)$.
- 14 We'll definitely include this in the appendix.
- Reviewer 2: Lipschitzness. The fairness and coverage constraints in Section 2 are Lipschitz in the confusion matrix, and so are the H-mean, Q-mean and Min-max metrics. For the G-mean and KLD metrics, we can easily construct close-approximations that are Lipschitz. We'll include these details in the paper, along with an example. As for the parameter λ , in theory it is sufficient to set it to a large-enough value as specified in Lemma 7. In practice, we set $\lambda = 10$, but the results were robust to changes in λ . Thanks for the suggestions to improve the writing and pointing out the typos!
- Reviewer 3: Limitations of a pre-fixed classifier. We agree that the performance of a plug-in classifier depends on the quality of the base class probability model. As an alternative, one can always train a new classifier from scratch in each step of Algorithm 1 to solve the linear minimization (LMO) over \mathcal{C} (line 5). This amounts to solving a cost-sensitive learning problem at each step. While the modified algorithm will be computationally more expensive, it no longer depends on a pre-trained model. Moreover, the number of calls to the LMO routine will be similar to Theorem 1, with the LMO-approximation term now depending on the quality of the classifier learned at each step. We'll include a discussion on this in the paper.
- Reviewer 4: Novelty. While we agree that the paper combines ideas from prior works, our main contribution is the re-formulation of a constrained classification problem as an optimization problem over the *intersection of two sets* $C \cap \mathcal{F}$, and the novel application of results from Gidel et al. (2018) [13] to solve the resulting optimization. This allows us to provide a new learning algorithm which (i) has a simpler structure than the previous algorithm, (ii) enjoys better convergence rate, (iii) can better handle non-smooth constraints, and (iv) is more robust to choices of hyper-parameters.
- Moreover, the proofs don't directly follow from the previous papers for the following reasons: (i) Gidel et al. provide an optimization algorithm, which does not directly apply to a statistical ML setup. For example, their proofs assume an exact LMO, whereas we had to explicitly take into account the error due to finite sample, including in their so-called fundamental descent lemma. (ii) Gidel et al. only provide a bound on the duality gap for the constrained optimization problem; we convert this into a bound on the sub-optimality and infeasibility of the learned classifier.
- Finally, we are able to handle a broader class of learning problems than Narasimhan (2018) [26], where the performance metrics can be defined by functions of more general "confusion vectors", which can depend on the instance x in more intricate ways.
- Reviewer 6: Convexity. We require the objective and constraints to be convex in the confusion matrix. We don't see this as a strong requirement as it is satisfied by all the example metrics in Section 2, including common fairness metrics such as equal opportunity and equalized odds. Yes, $B(\mathbf{u}, r)$ is a ball of radius r centered at \mathbf{u} ; Δ_n is the n-dimensional simplex. We'll make these notations clear.

Reference

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1. N. Cesa-Bianchi and D. Haussler. A graph-theoretic generalization of the Sauer-Shelah lemma. *Discrete Applied Mathematics*, 86(1):27–35, 1998.