

1 We thank all reviewers for their valuable comments. We will address all the minor points (typos, notation issues, and
2 remarks) as underlined and requested by the referees. Hereafter, we address comments shared by several reviewers.

3 **Generalization beyond k NN method.** First of all, although we apply our methodology to the k NN algorithm, we
4 provide general consistency results in Theorem 3.2 for the proposed plug-in procedure that apply to *any* off-the-shelf
5 estimators on the regression and the variance functions provided that these estimates are consistent. We illustrate this
6 capacity in particular in the numerical experiments on popular machine learning algorithms such as random forests or
7 svm for instance.

8 **Additional technical aspect.** As just mentioned, our methodology can be used to any estimators of the regression and
9 the variance functions. However, our general Theorem 3.2 asks for the continuity of the cumulative distribution function
10 of $\hat{\sigma}(X)$ (see Assumption 3.1). This assumption is violated when k NN algorithm is used and this is why, in addition
11 to its popularity, we specify our methodology in the context of k NN. In particular Section 4.2 presents a randomized
12 predictor based on k NN algorithm. This randomization technique used to circumvent Assumption 3.1 is rather simple
13 but we believe that it is instructive from a methodological aspect. Even if the considered estimators of the variance
14 function satisfies Assumption 3.1 the randomization technique can also be applied. In some sense, the construction
15 given in Section 4.2 makes our results more general. Besides, we mention that Assumption 3.1 is one of the limitation
16 that appears in Denis and Hebiri (2019) in the classification with reject option framework, and then Section 4.2 provides
17 a way to handle this issue.

18 **Extension to the regression setting with application to k NN.** We totally agree with **Reviewer 1**: most of our results
19 have direct analogues for classification (we will add more precise pointers to the final version upon acceptance).
20 Considering the regression setting is an interesting application of the reject option problem which is new in the literature
21 up to our knowledge and actually helps to understand a bit more the different characteristics of the model that are
22 relevant for the rejection rule such as the variance function (which is intuitive in the end but good to be noticed).
23 Moreover Section 4 is an original application to k NN that has no analogue and poses some additional technical tools
24 since *i*) Assumption 3.1 does not hold with k NN and then randomization is required; *ii*) it asks for a finite sample bound
25 on the L_∞ -norm estimation error of k NN which we derive based on previous asymptotic works on k NN.

26 **L_∞ -norm bound.** The obtained rates of convergence relies on Proposition C.1 (given in the supplementary material),
27 it requires rate of convergence for the estimation of the variance function with respect to the L_∞ -norm. In particular,
28 we establish that

$$\mathbb{E} \left[\left(\sup_{x \in \mathcal{C}} |\hat{\sigma}^2(x) - \sigma^2(x)| \right) \right] \leq C \log(n) n^{-1/(d+2)} .$$

29 This result relies on the rate of convergence of \hat{f} (the estimator of the regression function) *w.r.t.* L_∞ norm and can be
30 easily extended for instance to the class of partitioning estimators or kernel estimators, under similar assumptions as in
31 Section 4. That is to say the rate of convergence given in Theorem 4.4 applies to estimators which can be written as
32 $\hat{f}(x) = \sum_{i=1}^n W_{n,i}(x) Y_i$ where $W_{n,i}$ are weight functions. We also want to point out that we do not find a generic
33 reference for the rate of convergence for k NN estimates in L_∞ -norm. But, the proof of this result shares ideas similar
34 to the proof of Theorem 12.1 in Biau and Devroye (2015) which establishes only the consistency of the k NN estimates
35 *w.r.t.* L_∞ -norm.

36 **About the sample size of the semi-supervised procedure.** First, we want to notice that the result provided in
37 Theorem 4.4 shows, from a theoretical perspective, the dependency with respect to the sample size of labeled and
38 unlabeled sample. From the practical point of view, moderate values of N ($N \propto 100$) already describe a regime of
39 convergence for the estimation of $F_{\hat{\sigma}}$. We are convinced that numerical performance of the proposed semi-supervised
40 method is mainly dictated by the quality of the estimation of the regression function and the variance function which is
41 one of the main issues of the regression with reject option problem.

42 **Bibliographic remark.** Finally, we will include the references to the classification with reject option shared by the
43 referees. In particular, we will add in the text the reference to the paper of Herbei and Wegkamp (2006) to underline
44 the relation with the classification with reject option framework. We also want to highlight that, since we focus on the
45 regression setting, the variance function σ^2 is not necessarily bounded and thus we do not specify any upper bound for
46 λ in the paper.