First, we would like to thank R1, R2 and R3 for their evaluation, criticism and detailed comments which will help us improve the quality of our paper. We were pleased that R1 and R2 found the paper "very well written" and enjoyed the reading. We also are glad that R1 and R2 appreciated the "creativity" and the "beauty" of our hypothesis. We would finally like to thank R1, R2 and R3 for the mentioned related works and textual corrections; we will make sure to include all of these.

Answer to R1. Thanks for the great questions and suggestions. As R1 points out – our paper indeed focuses purely on "games", not on general "strategic interactions". Exactly because of this restriction, we are able to define and reason about a specific subclass of problems, that have interesting properties we investigate. Note though that it is important to understand that board games are not considered toy problems in the multi-agent learning and agents research community. They pose big challenges for machine learning algorithms, cfr. recent result in for example Go and Chess. They are effectively interesting abstractions of real-world situations, which are still complex but can be studied in a controlled setting. Games from Game Theory on the other hand are often focusing on very distilled, single high level challenge (e.g. RPS, social dilemmas), that abstract away a lot of dynamics. They are typically the first benchmarks to try out new multi-agent RL algorithms. We focus on providing a better understanding, and tools, for those working on what we describe as "real world games" (e.g. StarCraft, DOTA, Quake, Soccer, Go, etc.). We will add a statement explicitly stating the converse (that these properties will not necessarily transfer outside of this scope) in the camera ready version of the paper.

While Theorem 1 indeed shows that many games in EFG (e.g. Appendix L: "Random Games of Skill") will have extremely long cycles, this is not what constitutes the spinning top geometry. This is only one of the properties, that a spinning top like game would have - extremely long cycles around a very weak transitive component. However, many games do not follow this structure (e.g. Kuhn Poker, RPS etc.), thus clearly it is a non trivial class of games. We indeed claim that interesting, strategically complex, competitive, zero-sum, 2-player games imply this sort of geometry, but not the opposite implication (that only interesting games have this geometry).

To summarise, the main lessons learned, that can guide future research are as follows: we identify a set of geometrical properties, that real world games (defined as games created for human's enjoyment, in the form of competitive, strategically deep 2 player games), seem to share. While we were not able to prove theoretically the exact shape (and thus call it a spinning top *hypothesis*), we took a path from other empirical sciences - to form a hypothesis, and even if it is not directly provable - try to list various predictions it would make, and prove them instead: existence of very long cycles around the low transitive score, population size relation to convergence (that was observed in practice in research focusing on such games), and with empirical probing - we were able to see this shape emerging, while lacking in some other games created in game theory literature (thus confirming that the definition is not degenerate). On the theoretical side, we believe we pose a challenging open question of further structuring and formalising geometries of these types of games which, while of interest to the scientific community, so far have been often treated purely as "2 player symmetric zero-sum games". On the practical side, this research provides insights into population-based learning dynamics, required sizes of populations (which are much higher than those currently employed in practical applications), and suggests that in order to tackle these non-transitive challenges one might need more structured, latent policies, where a single set of weights can represent exponentially many types of behaviour (e.g. AlphaStar league "z conditioned agent"). Finally, we believe that these insights allow to construct strategically deep environments for AI to train on, as part of the current efforts of Open Ended Learning, and environment-agents co-evolution efforts. Ensuring the geometry of a spinning top (by creating cyclic interactions between actions on per time stamp basis while preserving skill dimension) can help AI researchers who are bottlenecked by not deep enough problems.

Answer to R2. First, we would like to thank the reviewer for the very positive feedback and great suggestions. In terms of proposition 2 we assumed 15hz control (used in e.g. Quake III Capture the Flag project). We could indeed provide bounds for other games, however for board games the bounding is a bit harder (since each move affects legality of future moves), thus Proposition 1 uses a subset of legal moves to provide the lower bound. The overall construction is algorithmicised and provided in the Appendix, and relies on being able to traverse the game tree, potentially with some heuristic choice of subset of actions in each node. We aim to provide bounds for other games as well in the camera ready version. For 3x3 board games we should be able to compute exact n.

Answer to R3. While we fully agree that there is a huge value in papers providing new algorithmic solutions for game playing agents, we also argue that the significance of understanding the underlying scientific phenomena of learning is equally important. Our paper situates itself exactly in this spectrum, where we aim to provide a better understanding and uncover a previously unknown geometry of the type of games we investigate.

We fully agree with the reviewer, that the problem we are tackling lies exactly in the intersection of classical RL, Game Theory, and Game Design. Unfortunately, all these three communities have independently built different naming conventions for the same objects. Consequently we spent a lot of time trying to unify, and present everything in a way that is possible to parse independently from the background.