

1 Thank you for all your comments. Our responses are detailed below, and we will incorporate them in the final paper.

2 **Reviewer 1: Variable number of comparisons k .** To simplify notation and analysis, it's standard in the BTL literature
3 to assume that pairs are compared a fixed number k times (see [10] or [12]). As explained in lines 150-152, "if k varies
4 between players so that i and j play $k_{i,j} = k_{j,i}$ games," the data Z can be normalized accordingly for our algorithm.
5 The bandwidth h can be altered, e.g., to use $h' = \min_{\{i,j\} \in \mathcal{G}(n,p)} k_{i,j}$ in place of k in equation (8). This would yield
6 corresponding theoretical guarantees with h' . We will clarify this in the final paper. One could also carry out the long
7 analysis in [10] (used in Lemma 3) and our analysis while keeping track of the $k_{i,j}$'s, and derive a corresponding h that
8 yields finer bounds. We decided to omit this due to space constraints.

9 **Reviewer 1: Ad-hoc steps in experiments.** We have already explained all the details needed to run all our experiments
10 in the paper. Perhaps the reviewer missed these details. Please see lines 150-151 and 238-246 for the precise values and
11 choices used to execute our algorithm, e.g., $h = 0.3n^{-1/4}$. On the other hand, we had not mentioned that we chose the
12 constant 0.3 in h , and the level of smoothing ("... game is counted as 20 games ..."), by eyeballing when the densities
13 in Figure 1 looked 'smooth.' Moreover, our qualitative results and trends in section 4 remain the same for a range of
14 values around 0.3 and 20 (e.g., 0.4 or 30). We will clarify these points in the final paper.

15 **Reviewer 1: Future directions and minor comments.** The minor corrections and clarifications will be made in the
16 final paper. Thank you very much for suggesting future research avenues pertaining to Bayesian or MDL approaches
17 and density estimation based on mutual fund performances.

18 **Reviewer 2: Interpreting skill PDFs.** We assume a lower bound on our skill PDFs over $[\delta, 1]$ in a neighborhood of 1
19 (see line 114). This implies that $\max_i \alpha_i \approx 1$ with high probability for large n (see (31) in supplementary materials).
20 Intuitively, we are re-normalizing all skill parameters so that the maximum one is essentially 1. So, if there are just two
21 teams with skills δ and 2δ , these values will be re-normalized to 0.5 and 1. Since $\max_i \alpha_i \approx 1$, it is reasonable for the
22 uniform skill PDF to put more mass on larger intervals, i.e., the uniform skill PDF is interpretable. Thus, we do not see
23 any immediate advantage of using logits. In Figures 1c and 1f, "World" and "English" have different skill PDFs but
24 similar skill scores, because different PDFs can have the same KL divergence to the uniform PDF. This artifact remains
25 even if we use logits. On a separate note, since the logits $\omega_i = \log(\alpha_i)$ are i.i.d. with PDF $P_\omega(t) = e^t P_\alpha(e^t)$, when δ
26 is constant and $h < \delta$, we have an estimator $\hat{\mathcal{P}}_2^*(t) = e^t \hat{\mathcal{P}}^*(e^t)$ for P_ω if desired. By substitution, $\mathbb{E}[\int (\hat{\mathcal{P}}_2^* - P_\omega)^2]$
27 $\leq (1 + \delta) \mathbb{E}[\int (\hat{\mathcal{P}}^* - P_\alpha)^2]$. Therefore, the upper bound in Theorem 3 also holds for MSE estimation of logit PDFs.

28 **Reviewers 3 & 4: Why not estimate skill score directly from $\hat{\alpha}_1, \dots, \hat{\alpha}_n$ (instead of estimating skill PDF P_α)?**

29 We outline several reasons to estimate P_α : (i) If one seeks to estimate a specific functional of P_α , e.g., moments or
30 variance, it is possible to estimate this directly from $\hat{\alpha}_1, \dots, \hat{\alpha}_n$. (This is still nontrivial because careful analysis is
31 needed to prove consistency of estimation based on noisy pairwise comparisons.) However, P_α contains information
32 about all such functionals and provides a lot more qualitative information as shown in Figure 1. Since different
33 functionals are needed for different applications, a good estimate of P_α rather than just samples is very useful. Our
34 main contribution is showing that the entire smooth density P_α can be estimated from $\hat{\alpha}_1, \dots, \hat{\alpha}_n$ as well as if we had
35 access to the true $\alpha_1, \dots, \alpha_n$. (ii) The dual characterization of TV distance and the Cauchy-Schwarz inequality give:

36 $T \triangleq \sup_{P_\alpha, \|f\|_\infty \leq 1} \mathbb{E}[(\int f d\hat{\mathcal{P}}^* - \int f dP_\alpha)^2] \leq \sup_{P_\alpha} \mathbb{E}[(\int |\hat{\mathcal{P}}^* - P_\alpha|)^2] \leq 3 \sup_{P_\alpha} \mathbb{E}[\int (\hat{\mathcal{P}}^* - P_\alpha)^2]$, where the first
37 sup is over P_α and all functions f bounded by 1. Thus, the bound in Theorem 3 holds for T . So, by estimating P_α , we

38 obtain *uniform guarantees on estimating any bounded statistic* of the form $\mathbb{E}[f(\alpha)]$, which includes all moments. (iii)
39 We believe differential entropy $h(P_\alpha)$ is an excellent overall skill score, and standard non-parametric estimators for it in
40 the literature (e.g., integral, resubstitution, or splitting data estimators) *require an estimate of P_α first to plug in*. (Also,
41 quantization theory shows that discrete "entropy of the empirical distribution" of $\hat{\alpha}_1, \dots, \hat{\alpha}_n$, with uniform binning, is
42 a poorer estimate of $h(P_\alpha) - \log(\text{bin size})$.) (iv) Philosophically, we believe that skill levels of players, like height or
43 weight, exhibit a distribution of values, and a tournament contains samples from this distribution. Hence, the "right"
44 skill score is based on P_α rather than the realizations $\alpha_1, \dots, \alpha_n$. We will elaborate on these points in the final paper.

45 **Reviewer 3: "Skill" vs. "exciting" vs. "competitive".** We will clarify that the differential entropy based "overall skill
46 score" measures variation of skills, not the actual skills of players, in the final paper. We used "exciting" sparingly in the
47 paper, but agree that it may not precisely capture what we mean. So, we will change or clarify it in technical discussions
48 in the final paper. We have also sparingly referred to tournaments with high overall skill scores as "competitive,"
49 because many teams have similar skill parameters and game outcomes are less predictable. This usage seems reasonable
50 and we have retained it. We also agree that closely matched teams may have many non-competitive games, and our
51 overall skill score is indeed an "average measure" which may not capture these low probability events.

52 **Reviewer 4: Theorem statements.** Using the phrase "sufficiently large" is *standard practice* in mathematical statistics
53 (see, e.g., [10]), and it has a very *precise meaning*. "Sufficiently large n (or constant c)" means that "there exists a
54 constant A such that for all $n \geq A$ (or $c \geq A$)." Here, the values of A may depend on other constant problem parameters,
55 and they can be deduced from our proofs. For example, in Theorem 1, the constant $c_{15} \geq 2c_4/(\delta\sqrt{pk})$, where c_4 is the
56 universal constant from Lemma 3 (which is Theorem 3.1 in [10]). Moreover, for "large constants," we already mention
57 which parameters A depends on in the paper. Since we do not derive sharp values of A , it is not illustrative to include
58 them in theorem statements. However, our theorem statements are rigorous; e.g., they directly imply big- O style results.