

1 We thank all four reviewers for their encouraging remarks and thorough treatment of our work. We respond individually  
2 to Reviewers 1 and 2, and jointly to Reviewers 3 and 4.

3 **Reviewer 1 Response:** We thank Reviewer 1 for their in-depth comments and questions on the supplementary materials.  
4 We address questions below. Smaller comment/questions not addressed will still be updated in the final version.

5 “*Is there really a need to present Alg 1 and Thm 3.5?*” Alg 1, Thm 3.5, and Thm 3.6 are present in order to show a tight  
6 characterization of the query complexity of Comparison-Pool-PAC learning. While it is true that Theorem 4.11 also  
7 implies an upper bound on the weaker Comparison-Pool-PAC model, it leaves an unnecessary  $\log(1/\varepsilon)$  gap between  
8 upper and lower bounds which Theorem 3.6 closes.

9 “*In Alg 1, Threshold( $S$ ) is only informally defined, and an elaboration is needed. I think basically, the algorithm can  
10 successfully approximately recover  $b$  if it can find two neighboring + and - examples?*” The procedure Threshold( $S$ )  
11 produces some threshold consistent with labeling  $S$  by binary search (note that this procedure may mislabel small  
12 portions of  $S$  near the true threshold). We will clarify this in the text. In Theorem 3.6, we show this procedure produces  
13 a good threshold with probability at least  $2/3$  based on neighboring  $+/-$  examples. This is later amplified by Chernoff.

14 “*In the second half of Corollary 4.7, is the goal only to identify the labels of all examples  $S$  drawn? Also, what is the  
15 active learning algorithm used here?*” Yes, the second half Corollary 4.7 only finds the labels of  $S$ . The algorithm is  
16 [KLMZ17](Theorem 3.2) (Alg box pg. 16). In the text, we restate this theorem as Theorem 2.1, and reference it in the  
17 proof of Corollary 4.7. We will add further description of [KLMZ17]’s algorithm and why it makes no errors when  
18 assuming the wrong inference dimension (this is because inference dimension controls only the algorithms coverage).

19 “*In Theorem 4.9, is  $H$  here  $H_{d,\eta}$ ? ...What is  $h \times [-\gamma, \gamma]$ ?*” Yes, in Theorem 4.9  $H$  should be  $H_{d,\eta}$ , and  $h \times [-\gamma, \gamma]$  is  
20  $\{x : h(x) \in [-\gamma, \gamma]\}$ . We will update these.

21 “*[is it] possible that there exist two hypotheses  $h_1, h_2$  that have very small minimal-ratio, and they agree with the  
22 queries in  $Q(S'_h - \{x\})$ , but disagree on  $x$ ?*” Yes, this is possible for arbitrary  $h_1, h_2 \in H_d$ . However, this does not  
23 affect our argument, since we have reduced (with very high probability) to the case that  $h$  has large minimal ratio with  
24 respect to  $S'_h$ . Note that this does not cause any errors since it is not an assumption, but rather is based off of a verifiable  
25 structural property of  $S$  (no large subset is too close or too far from any  $h \in H_d$ ).

26 **Reviewer 2 Response:** We thank Reviewer 2 for their comments. While we agree with much of their assessment, we  
27 would like to address two aspects of the review with which we disagree.

28 **Computational Efficiency:** The Reviewer’s only stated weakness is that “*The paper does not consider computational  
29 efficiency and noise tolerance.*” While it is true that our work focuses mainly on characterizing the query complexity of  
30 realizable-case learning, the former part of this statement is false: we *do provide computationally efficient algorithms*  
31 for both the RPU and PAC models, and *explicitly state this* in the paper (lines 214, 295). In more detail, our main  
32 contribution, the  $\tilde{O}(d \log(1/\varepsilon)^2)$  Comparison-Pool-RPU learning upper bound, is computationally efficient, which also  
33 implies a computationally efficient algorithm for the strictly weaker PAC-model. That said, we realize that this fact  
34 is somewhat hidden in the paper, and thank the reviewer for bringing it to our attention. We will add a discussion of  
35 computational efficiency to the introduction to fix this.

36 **Novelty and Focus:** We would also like to clarify what we view as a misunderstanding of the focus and novelty of our  
37 work. Reviewer 2 comments mostly on our Comparison-Pool-Pac upper bound (Theorem 3.3) based upon [BL13], and  
38 at one point states “*the paper is built upon [BL13].*” In fact, [BL13] is used only once in the entire paper in order to help  
39 characterize the query complexity of Comparison-Pool-PAC learning. *We would like to highlight that we do not view  
40 this as the works’ main contribution or novelty.* Rather, as we state in the paper on lines 217, 251, and 277, the main  
41 novelty and focus of our work lies in the analysis of query and computationally efficient RPU-learning, and especially  
42 in the introduction and development of average inference dimension—a novel tool for analyzing distribution dependent  
43 RPU-learning crucial to these results. It is worth noting that Reviewer 1 asks why we even include the [BL13] based  
44 result given our stronger results on RPU-learning (the only reason is a  $\log(1/\varepsilon)$  gap in query complexity between the  
45 two algorithms).

46 **Reviewers 3 and 4 Response:** We thank Reviewers 3 and 4 for their encouraging comments, and believe both reviews  
47 appropriately frame and summarize our work. In response to Reviewer 4’s suggestion, should the paper be accepted we  
48 would be happy to use part of the additional camera-ready page for covering further research perspectives.

#### 49 References

50 [BL13] Long P. Balcan, M. Active and passive learning of linear separators under log-concave distributions. In  
51 *Proceedings of the 26th Conference on Learning Theory*, 2013.

52 [KLMZ17] Lovett S. Moran S. Zhang J. Kane, D. Active classification with comparison queries. In *IEEE 58th Annual  
53 Symposium on Foundations of Computer Science*, 2017.