

1 We thank the reviewers for their time, helpful feedback, and advice. We are pleased that overall, reviewers praised the
2 clarity, rigour and contributions of the work. We are encouraged that reviewers acknowledged novelty (R3, R4) and
3 appreciated our work as a principled contribution in the development of machine learning methods for non-Euclidean
4 data (R1, R2). We thank reviewers for their kind words, and hope to address any remaining concerns below.

5 **(R1, R2, R3) Uni-modal distributions.** We thank the reviewers for raising this question. The goal of our two
6 experiments with unimodal target distributions is to demonstrate specific pathologies of previously introduced methods.
7 We believe this is most clearly demonstrated by a unimodal distribution at the point (or the limit) of the pathology as it
8 removes additional modelling artefacts that are introduced through more complex distributions. We further evaluate the
9 capacity of our method to model highly multi-modal distributions in our real-world experiments in which we can show
10 substantial improvements to baselines. We will clarify the different purpose of these experiments in the paper.

11 We agree with (R2) that "the stereographic projection is known to lead to problems in its singular point." Yet, current
12 SOTA methods are actively using such a parametrization (e.g., Gemici et al. (2016)) and we believe it is important
13 to show and evaluate this aspect. Moreover, most manifolds of interest similarly have a non-Euclidean topology, so
14 applying previous *projected* methods on these manifolds would also yield a similar pathology.

15 **(R2, R3) Real-world data.** Our experiments on real-world data are motivated from problems in climate and earth
16 science, hence leading to empirical assessments on \mathbb{S}^2 . We believe these experiments to be informative since they show
17 that our method is a) scalable and b) can fit highly complex and multi-modal distributions more accurately than previous
18 methods. Moreover, please note that our model can straightforwardly be applied on higher dimensional manifolds.
19 We agree with reviewers that our model would be strengthened by an additional experiment on a high dimensional
20 manifold. To achieve this we will run a fourth experiment by first computing hyperbolic multi-dimensional embeddings
21 of WordNet graph data, and then fitting our model to the obtained empirical distribution.

22 **(R4) Optimal transport (OT) and flows.** As reminded by (R4), the field of OT is core for the study of flow – known
23 as *transport map* – transforming one distribution into another. Indeed, we develop this connection in Appendix D.1
24 and show that the dynamical formulation of OT still holds in the manifold setting. Regarding (R4)'s concern that our
25 method "can not model the transformation for a white noise to multi-mode distributions": Our method is theoretically
26 sound as it able to model multi-modal distributions as long as supports are connected (see Cornish et al. (2019, Theorem
27 2.1)), and is empirically shown to model well multi-modal earth data (cf Table 3 and Figure 6).

28 **(R1, R2) Computational aspects.** We thank (R1) for suggesting to expand on the limitations of our method. As
29 reminded by (R2), projections required by our approach can increase the computational cost. With our current
30 implementation, we empirically find that this additional cost amounts to $\sim 20\%$ for the Poincaré ball and $\sim 30\%$ for
31 \mathbb{S}^2 . This cost can be further reduced by only projecting the output of the ODE solver steps. We thank (R2) for the
32 suggestion and we will rigorously compute and include wall-clock time comparisons in the next draft.

33 **(R1, R2) Empirical comparison to related methods.** We thank (R2) for suggesting the mixture of von Mises-Fisher
34 (vMF) baseline. Indeed, we found in our early empirical assessments that high multi-modality (e.g., as occurring in the
35 different the earth datasets) would prevent a mixture of vMF distributions from being a competitive baseline. We will
36 re-run this baseline and include its performance in Table 3.

37 We also agree with (R1) that comparing our method to Rezende et al. (2020) would indeed be valuable. Unfortunately,
38 the code and necessary experimental details have not yet been released, preventing us from a detailed comparison.

39 **(R2, R4) Additional manifolds.** We agree with (R4) that applying the proposed method to more exotic manifolds
40 is an exciting direction. We are currently exploring applications on several Lie groups such as orthogonal or positive
41 definite matrices. As reminded by (R2) we indeed assume that the manifold is known beforehand. We agree that
42 learning the manifold is an exciting task, but indeed a harder one as shown by the field of topological data analysis.

43 **(R2) VAE experiments.** We thank (R2) for suggesting to leverage our model in a VAE setting. We agree that this is
44 indeed a promising application of our method (e.g., see also Bose et al. (2020)) and plan to explore this in future work.

45 **(R2) Naive Euclidean method.** We thank (R2) for highlighting the potential risk for non-careful readers to compare
46 manifold-valued densities against \mathbb{R}^D valued ones. We updated the introduction to better refer this.

47 References

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