

1 Thanks for your insightful comments and for your recognition of the originality (reviewers 1, 2 & 3) and significance
2 (reviewers 1 & 2) of our paper. Below we respond to your comments.

3 **To Reviewer 1 & 2: Reorganization Plan.** We agree that the paper’s space management should be improved. Based
4 on your comments, we propose the following feasible reorganization plan: (1) provide more explanations on the
5 lower bound proof techniques; (2) increase Section 5 (general switching costs) to about 1.5 pages with more detailed
6 discussions of the HS-SE policy and Theorem 3 & 4; (3) revise the abstract & introduction to highlight the contributions
7 that would be discussed in detail in the main text; and (4) remove some secondary content.

8 **(1) Expanded proof sketch of Theorem 2.** For any $T \geq 1, k \geq 1$ and $S \geq 0$, for any S -switch policy $\pi \in \Pi_S$,
9 we want to find an environment \mathcal{D} such that $R_{\mathcal{D}}^{\pi}(T)$ is larger than the desired lower bound. A key challenge here
10 is that π is an arbitrary and abstract S -switch policy — we need more information about π to construct \mathcal{D} . With
11 this goal in mind, we first design a concrete “primal environment” α . We use this environment to evaluate policy
12 π , such that we can observe some key patterns revealed by policy π under α . These patterns are characterized by a
13 series of ordered stopping times $\tau_1 \leq \tau_2 \leq \dots \leq \tau_{m(S)+1}$, some of which may be ∞ , that are recursively defined
14 as follows: add line 197-200 of the article. We then compare the realization of $\tau_1, \dots, \tau_{m(S)}$ with a series of fixed
15 values $t_1, \dots, t_{m(S)}$, which are the endpoints of the intervals defined in Algorithm 1. Based on the possible outcomes
16 of comparisons, we define $m(S) + 1$ key events: add line 9-11 in Appendix page 5, at least one of which must occur
17 under π and α with probability at least $1/(m(S) + 1)$. We then do a case by case analysis as follows. In the first
18 case, $\{\tau_1 > t_1\}$ occurs with certain probability, indicating that the action chosen in round τ_1 was not chosen in $[1 : t_1]$
19 with certain probability; in the second case, $\exists j \in [2 : m(S)]$ such that $\{\tau_{j-1} \leq t_{j-1}, \tau_j > t_j\}$ occurs with certain
20 probability, indicating that the action chosen in round τ_j was not chosen in $[t_{j-1} : t_j]$ with certain probability; in the
21 third case, $\{\tau_{m(S)} \leq t_{m(S)}\}$ with certain probability, indicating that the number of switches occurs in $[t_{m(S)} : T]$
22 is at most $k - 1$. For each case, we construct an “auxiliary environment” β by carefully adjusting α based on the
23 aforementioned indication. The environment β ensures two things: (i) β is “hard for π to distinguish from α ”, such that
24 a crucial event (constructed based on the indication) that occurs under π and α with certain probability also occurs under
25 π and β with similar probability; and (ii) β is “different enough from α ” such that the certain occurrence probability of
26 this event under β makes $R_{\beta}^{\pi}(T)$ larger than the desired lower bound. Theorem 2 then follows by $R^{\pi}(T) \geq R_{\beta}^{\pi}(T)$.

27 **(2) Increasing Section 5.** First, we will have a lengthier discussion of the HS-SE policy. We emphasize and explain two
28 new ingredients of the HS-SE policy that are not in the SS-SE policy. (i) A pre-specified switching order: within each
29 interval, the HS-SE policy switches based on an order determined by the shortest Hamiltonian path of the switching
30 graph G ; (ii) A reversing policy: the HS-SE policy switches along one direction in the odd intervals, and along the
31 reverse direction in the even intervals. We then illustrate how (i) and (ii) enable the HS-SE policy to repeatedly visit
32 all effective actions in an economical way to stay within budget, and how this motivates Theorem 3. Second, we will
33 provide insights on how we extend the unit-cost lower bound to the general-cost lower bound, by highlighting an
34 important step of the proof, which is to let $(\arg \max_i \min_{j \neq i} c_{i,j})$ be the optimal action in the “primal environment” α
35 so that switching from and to this action is costly. Third, we briefly discuss the implications of the bounds.

36 **(3) Revising abstract & introduction.** We will adjust the abstract and introduction to clarify the focus and scope of
37 the paper. In particular, we will highlight the following contributions: (i) the SS-SE policy, the lower bound proof, and
38 phase transitions and cyclic phenomena in the unit-cost setting; and (ii) the HS-SE policy and extended bounds in the
39 general-cost setting, with a surprising connection to the shortest Hamiltonian path problem.

40 **(4) Removing some content.** We defer Figure 1 (as Table 1 is enough), Section 4.3.2, and Section 4.4 to the appendix,
41 and greatly shorten line 65-85 & 115-131. Then we have enough space to expand the proof sketch and Section 5.

42 **To Reviewer 1.** (1) We agree that given the related literature in the adversarial MAB, a “conceptual” relationship
43 between BwSC and the batched bandit problem in the stochastic setting may be expected. However, what we discover is
44 a precise relationship — a “regret equivalence” between S -switch k -armed BwSC and the M -batched k -armed bandit
45 problem, see line 275-278. That is, there is an explicit formula $M = \lfloor \frac{S-1}{k-1} \rfloor$ directly translating the regret bounds and
46 the optimal algorithms of the two problems. While this is a surprise to us, we follow your suggestion and defer Section
47 4.4 to the appendix. (2) If the shortest Hamiltonian path is only approximately computed in Algorithm 3, say we have a
48 Hamiltonian path of length δH ($\delta > 1$) instead of the shortest Hamiltonian path of length H , then the index $m_G^U(S)$ in
49 the upper bound in Theorem 3 becomes $\lfloor \frac{S - \max_{i,j} c_{i,j}}{\delta H} \rfloor$. As long as S is not too small, the new upper bound is still
50 close to the lower bound in Theorem 4 — the gap between them decreases doubly exponentially with S .

51 **To Reviewer 2.** Yes, the “actual regret scaling” could be smoother than the “worst-case regret scaling”. Note that phase
52 transitions are associated with asymptotic bounds of the worst-case regret, so if (1) the underlying distributions are not
53 the worst-case distributions and we are focusing on the “actual incurred regret”, or (2) T is not large enough to dominate
54 the constants in the bounds, phase transitions may not be exhibited. We will highlight point (1) and (2) in Section 4.3.1.

55 **To Reviewer 3.** Following your suggestion, we conducted computational experiments in the setting of $k = 3, S =$
56 $1, \dots, 6$ and $T = 10^3, 2 \times 10^3, \dots, 10^4$. Running the SS-SE policy under several sets of underlying distributions, we,
57 as expected, observe the cyclic phenomena of the incurred regret. However, it is computationally expensive to show the
58 cyclic phenomena of the worst-case regret of other policies, as this requires iterating over all possible distributions.