Copula Processes Supplementary Material

In this supplement, we briefly expand on some points in the main text of our paper.

Gaussian Copula Covariance Matrix

For the Gaussian Copula in (9), the prior $\Gamma_{ij} = \text{cov}(g^{-1}(\sigma_i), g^{-1}(\sigma_j)) = \text{cov}(f_i, f_j) = k(t_i, t_j)$. The posterior Γ_{ij} can be estimated as the covariance matrix of the Laplace approximation for $p(\boldsymbol{f}|\boldsymbol{y})$. Also, since each component of $\boldsymbol{f}$ is transformed separately, such that $\sigma(t_i) = g(f(t_i))$, we have

$$p(\boldsymbol{\sigma}|\boldsymbol{y}, \boldsymbol{z}) = [\prod_{i=1}^{N} \frac{df_i}{d\sigma_i}] p(\boldsymbol{f}|\boldsymbol{y}, \boldsymbol{z}) = [\prod_{i=1}^{N} \frac{1}{g'(f_i, \boldsymbol{\omega})}] p(\boldsymbol{f}|\boldsymbol{y}, \boldsymbol{z}). \tag{1}$$

One can use this to simulate from the joint distribution over the deviations.

Laplace Approximation

A comment about our modification to Newton's method:

At a maximum, the negative Hessian of the objective function, $W + K^{-1}$, is positive definite. On each iteration of Newton's method, we form M by setting all negative entries of W to zero. Since K^{-1} is positive definite, and the eigenvalues of $M + K^{-1}$ are greater than or equal to the eigenvalues of K^{-1}, $M + K^{-1}$ is always positive definite. Using M in place of W decreases the Newton step size, and changes the direction of steps.

Also, $B = I + M^{\frac{1}{2}} K M^{\frac{1}{2}}$, is often well conditioned, since it has eigenvalues no smaller than 1, and no larger than $1 + n \max_{ij}(K_{ij})/4$.