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# Quantifying Spike Similarity through EMD

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As a cross-bin similarity measure, which can be robust to waveform misalignment, the Earth Mover's Distance (EMD) [1, 2] gains its name from the intuition that given two waveforms, one can be seen as a mass of earth properly spread in space, the other as a collection of holes in that same space. The EMD measures the least amount of work needed to fill all of the holes with all of the earth, where a unit of work corresponds to transporting a unit of earth by a unit of ground distance. In this paper, we formulate the EMD in the specific context of spike sorting, where the EMD is employed to compare a spike template (object model) and a spike waveform (object candidate). Specifically, we denote the ground distance between the  $u^{th}$  sample in the object model and the  $v^{th}$  sample in the object candidate as  $d_{uv}$  (e.g. Euclidean distance  $d_{uv} = |u - v|$ ), and the flow (amount of transported earth) between them as  $f_{uv}$ . The goal is to find the best alignment  $t$  that corresponds to the smallest EMD

$$\arg \min_t (\min_{f_{uv}} Z(f_{uv}(t))). \quad (1)$$

In Eq.1 the inner optimization is to find the EMD for each alignment, and the outer one is to obtain the best alignment. In the following, we use the superscript  $M$  to denote the object model and  $C$  for the object candidate.  $w_v^{(C)}$  is the weight of the  $v^{th}$  sample in the object candidate and  $w_u^{(M)}$  the weight of the  $u^{th}$  sample in the object model, respectively. As EMD works most conveniently on waveforms with non-negative, equally summed weights ( $w_v^{(C)}$  and  $w_u^{(M)}$ ), both object candidate and object model are aligned to the same DC level with all the samples being positive

$$w_i(n) = V_{spike}(n) - \frac{1}{N_{spike}} \sum_{n=1}^{N_{spike}} V_{spike}(n) + V_{DC}, \quad (2)$$

where  $w_i$  represents an object candidate/model,  $V_{spike}$  represents a spike waveform under similarity measure, and  $V_{DC}$  is an arbitrary DC bias to satisfy  $w_i(n) > 0, \forall i, n$ . ss According to the definition of EMD [1],  $Z$  in Eq. 1 is formulated as

$$Z(f_{uv}(t)) = \sum_{u=1}^{m^{(M)}} \sum_{v=1}^{m^{(C)}} d_{uv} f_{uv}(t),$$

subject to

$$\begin{aligned} \sum_{u=1}^{m^{(M)}} f_{uv}(t) &= w_v^{(C)}(t), \quad 1 \leq v \leq m^{(C)} \\ \sum_{v=1}^{m^{(C)}} f_{uv}(t) &= w_u^{(M)}, \quad 1 \leq u \leq m^{(M)} \\ \sum_{u=1}^{m^{(M)}} \sum_{v=1}^{m^{(C)}} f_{uv}(t) &= N_{spike} V_{DC} \\ f_{uv}(t) &\geq 0, \quad 1 \leq u \leq m^{(M)}, 1 \leq v \leq m^{(C)}. \end{aligned}$$

Equation 1 as a linear programming problem can be considered in geometric terms as finding an optimum in a closed convex polytope. In the problem presented in this work, the polytope is defined by intersecting  $m^{(M)} + m^{(C)} + 1$  half-spaces in a  $m^{(M)} \times m^{(C)}$ -dimensional Euclidean space. Computing the EMD is based on a solution to the wellknown transportation problem [3] from linear optimization, for which efficient algorithms, e.g., simplex methods, are available. The simplex method essentially works by searching the boundary of the polytope for an optimum. Detailed descriptions of the simplex method to solve Eq. 1 are presented in [4, 5]. In the experiment section, cluster isolation quality is quantitatively scored by EMD based similarity measure.

## References

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