

1 **R#1:** Thanks for stressing the strengths of the paper (a complete theory of FP in MFG and a rich empirical evaluation). We  
2 first address the stated weaknesses. **W1: Short presentation of FP.** We'll improve it for the final version by adding  
3 formal def of the best response and explaining why an arbitrary policy between  $[0, 1]$  is needed for init purpose. **W2:**  
4 **Gap between CTFP and practical algs:** We'll add the following discussion to the paper. We chose to provide an  
5 analysis in continuous time because it provides convenient mathematical tools allowing to exhibit state of the art  
6 convergence rate. The convergence rate in discrete time is still an open problem even for 2-players games, but would  
7 be an interesting research question (there is a known conjecture in  $O(1/\sqrt{t})$  [75]). **Detailed comments: (1)** We  
8 acknowledge that some useful details should be moved from appx to the main text for the sake of clarity. E.g. the  
9 computation of the Best Response (BR) and the population distribution (*cf.* Appx) are both used in FP (Alg. 1), which  
10 is implemented in two different settings: a model-based and a model-free approach. The model-based uses Backward  
11 Induction (BI, Alg. 4) and an exact calculation of the population distribution (Alg. 5). The model-free approach uses  
12  $Q$ -learning (Alg. 2) and a sampling-based estimate of the distribution (Alg. 3). As suggested, we will add the update  
13 rules of the  $Q$ -function of both methods in the main text. We will clarify how the distribution  $\hat{\mu}_n^\pi$  (Alg. 3) is used  
14 in Alg. 2 by using proper notations.  $Q$ -learning and BI approximate the BR against  $\bar{\mu}^j$  (mean distribution), which  
15 needs to be clarified: we will add a line in Alg. 1  $\bar{\mu}^j = \frac{j-1}{j}\bar{\mu}^{j-1} + \frac{1}{j}\hat{\mu}^j$  (so here,  $\hat{\mu}_n^\pi$  and  $\hat{\mu}^j$  are the same). In Alg.  
16 2, the  $\mu$  of the input can be any distribution ( $\mu = (\mu_k)_k$ ) but we use the mean population distribution  $\bar{\mu}^j$  (from the  
17 previous FP step) in our setting. **(2) Randomization:** As we use  $\bar{\mu}^j$  we don't need to select the policy uniformly over  
18 previously obtained policies. Also, we already do employ randomized strategies (for the model-free), with  $\varepsilon$ -greedy  
19 exploration parameter set to 0.2 (1.140). Authors of [64] use a softmax to ensure the regularity needed in their proof.  
20 To the best of our understanding, Angiuli *et al.* use  $\varepsilon$ -greedy action because the updates of  $Q$  and  $\mu$  are intertwined,  
21 so the exploration/exploitation are mixed. In Alg.2, the  $Q$ -learning (with exploration) and the action to update the  
22 distribution (with pure exploitation) are separated. Furthermore, the stochasticity of the environment (noise  $\epsilon_n$ ) adds  
23 randomization. Note that randomization is not necessary in model-based as the BR and population distribution are  
24 computed exactly (which also bridges the gap between model-based and the theory). Adding  $\varepsilon$ -randomization or a  
25 softmax in the distribution update is an interesting direction. **Exploitability:** Please notice that, because it scales with  
26 rewards, its absolute value is not meaningful. This quantity is game dependent and hard to re-scale without introducing  
27 other issues (dependence on the initial policy if we re-normalize with the initial exploitability for example). But it  
28 decreases by a large factor compared with the initial value. **(3)** The problem of error propagation is addressed in [51]  
29 (see Eq. 7). However, [51] does not provide any rate for discrete time FP. As opposed to this work, we focused in  
30 getting a convergence rate for CTFP without approximations (in a wider set of settings than in [51]). Surprisingly, these  
31 rates do not seem to be too off in practice. We also introduce a new theory of common noise for the two practical algs  
32 (*c.f.* R#3). **(4)** We will improve on that transition stating that to go from continuous to discrete time we simply replace  
33 sums by integral and difference equations by differential equation (inclusion to be precised). The "watershed" region is  
34 necessary to make sure the differential equation is defined on a closed set (here  $[1, +\infty[$ ). Without it, we would only  
35 be able to define it on  $]0, +\infty[$  which is not enough. **(5)** We apologize for the too short Sec.3. We'll rewrite it with  
36 elements from appx A. Even if not directly used, we felt that the equation involving  $\pi_n^t$  was important as it is easier  
37 to manipulate policies compared to distribution over states. **(6)** Our common noise can be history dependent (i.e., no  
38 assumption on it). In the experiment of Sec.6, the common noise is stationary and i.i.d. Common noises affect the  
39 transition probability of the distribution, which is then *random* (it is not the case with only idiosyncratic noise).

40 **R#3:** We are grateful for the positive comments acknowledging the importance of common noise in MFGs and MARL,  
41 and on the fact that our contribution bridges the gap between MFG and tools from algorithmic game theory such as  
42 exploitability. **W1: connections with MARL examples:** Actually our numerical examples are strongly motivated by  
43 classical examples in the RL literature. For instance, the beach bar process example is a simplified version of the well  
44 known Santa Fe bar problem, which has received a strong interest in the MARL community, see e.g. [Farago et al, Fair  
45 and Efficient Solutions to the Santa Fe Bar Problem (2002)]. Similarly, the maze is motivated by swarm motion models  
46 from the distributed robotics MARL literature. We will stress this point and add references in the revision. **Other**  
47 **works:** Thank you for pointing out these relevant references, that we will cite as well. Note however that, compared  
48 with these works, our paper provides a rigorous rate of convergence, and covers the common noise setting. Last but not  
49 least, our work is not limited to potential or variational MFGs as we only need the weaker monotonicity assumption.

50 **R#5: (1)** We strongly disagree about the lack of novelty and incremental nature of our work, and would have appreciated  
51 some argument for this harsh comment. We would like to stress that the other two referees have acknowledged the  
52 novelty of work (rate of convergence, common noise, etc.). **(2)** The monotonicity assumption is classical in the MFG  
53 literature and much weaker than assumptions made in other works (regularity and smallness of the coefficients in [64],  
54 potential structure in [84], etc.). Also, R#1 considers these assumptions as mild. **(3)** This is the very principle of the  
55 fictitious play to obtain convergence for averaged policies. We would appreciate any reference where it is not the case.