

1 We thank all the reviewers for their detailed comments and suggestions.

2 **Review #2:**

3 1: Thanks for pointing this connection. Yes, it is a version of the exponential weights, where the losses are the gradients  
4 of the smoothed hinge loss. We will add this remark to the revision.

5 2: The smoothed loss is actually quite important to our algorithm. While one could likely use an algorithm with a  
6 second-order bound like Squint to learn the weights instead of our FTRL-based approach (which we chose mainly for  
7 simplicity), we believe that the losses provided to the algorithm must be the gradients of the *smoothed loss* rather than  
8 just the individual correlations of the hints. The reason we think the smoothed loss is necessary (as discussed in lines  
9 127-132) is that a linear combination of individually bad hints might produce a hint that is actually extremely good. It is  
10 not obvious how to capture this without using a loss that aggregates information from all hint sequences as our smooth  
11 loss does.

12 3: We agree with the suggestions and will incorporate them; thank you for the careful reading.

13 4, 5, 7, 8: We apologize for these typos and slight inaccuracies; we will fix them in the revision.

14 6: Yes, we assume  $\alpha \leq 1$ .

15 9: Our current proof is unable to show a high probability bound, but this is an interesting question, thanks! We will add  
16 this remark to the revision. We will also add the similarity of our analysis to quick-sort and paging.

17 **Review #3:**

18 Regarding the combiner algorithm and comparison to [Cutkosky 2019]: at first blush it is true that the [Cutkosky  
19 2019] combiner only requires small loss at the origin, but it is actually difficult to use it in constrained settings even  
20 when appealing to the black-box constraint set reduction proposed in [Cutkosky & Orabona 2018]. This is because the  
21 reduction changes the losses in a way that might damage the regret bounds of the algorithms that are being combined. In  
22 particular, the reduction requires one to commit to a particular norm, which makes it difficult to design an algorithm that  
23 combines base algorithms that use different  $p$  norms and is also constrained, as we are able to accomplish in Theorem  
24 16. In fact, even in [Cutkosky 2019], the constrained optimism algorithm requires an ad hoc technical modification to  
25 the constraint set reduction in order to work. Nevertheless, as you suggest, we will add a short discussion about these  
26 subtleties and the limitations of the prior work.

27 Quantifiers: We will add the missing quantifiers to theorem statements to make them clearer. As suggested, we will add  
28 the comparator  $u$  to the notation for regret in all appropriate places.

29 As you correctly notice, there is a small gap of  $\sqrt{(\ln T)/\alpha}$  in the bounds. We will add a discussion to this effect.

30 **Review #4:**

31 Our main algorithm has two parts: (i) an algorithm to find an optimal combination of different hint sequences for a  
32 known value of  $\alpha$ , and (ii) a combiner algorithm to deal with unknown  $\alpha$ . To the best of our knowledge, both involve  
33 novel techniques that potentially have broader applications.

34 (i) Smooth hinge loss: Since a combination of multiple hint sequences can be significantly superior to any of the  
35 individual hint sequences, using the individual correlations of the hints is not sufficient. We introduce a novel smoothed  
36 hinge loss precisely to deal with this issue and show that using FTRL on these new losses helps obtain a new hint  
37 sequence that can provide regret comparable to the best combination of the original hints.

38 (ii) Combiner algorithm: This is a new, general way to combine  $K$  online learners while obtaining regret that is as  
39 good as that of the best learner, to a factor  $\log K$ . The closest work we are aware of is [Cutkosky 2019, “Combining  
40 Online Learning Guarantees”], but our approach is conceptually quite different and applies in different settings (see our  
41 response to Review 3 for discussion of the differences). Furthermore, we show how the combiner can be used to obtain  
42 new results outside the setting of our problem. Appendix E contains two such applications: adapting to different norms  
43 and simultaneous Adagrad and dimension-free bounds.

44 In addition to adding these remarks, in the revision, we will position our algorithms and the proof techniques better  
45 with respect to related literature.