

1 **Reviewer 1: Q1:** This paper should provide experimental results?
2 A: We believe it is important to deliver the message that our proof is novel that addresses the open
3 problem for strongly-convex-strongly-concave minimization. Hence, we emphasize on theoretical
4 analysis in this paper. On the other hand, previous studies have provided the numerical experiments
5 on the state-of-the-art algorithms that are highly related to our Epoch-GDA, e.g., [36,32]. There is
6 also a following-up work that uses a similar idea and has promising experimental results [ref1]. We
7 will consider adding some experiments in the long version.

8 [ref1] Guo et al. “Fast Objective and Duality Gap Convergence for Non-convex Strongly-concave
9 Min-max Problems”. arXiv 2020.

10 **Q2:** Key difference of algorithm and analysis between the proposed Epoch-GDA and Epoch-GD?
11 A: The update of Epoch-GDA can be seen as a primal-dual variant of Epoch-GD. In terms of analysis,
12 there is key difference between Epoch-GDA and Epoch-GD. In particular, Epoch-GD bounds the
13 primal gap, while Epoch-GDA bounds the duality gap. Note that bounding the duality gap of a
14 min-max problem is fundamentally more difficult than bounding the primal gap of a minimization
15 problem. The difference between our analysis and that of Hazan & Kale is very subtle. Particularly,
16 in Hazan & Kale, they used the fact that the primal gap at a solution x_{k+1} from stage $k + 1$ can be
17 bounded by the distance between a solution x_k from stage k and the optimal solution i.e., $\|x_k - x_*\|$,
18 which can be further bounded by the primal gap at x_k using strong convexity. However, in our
19 case, the duality gap at a solution (x_{k+1}, y_{k+1}) from stage $k + 1$ cannot be bounded by the distance
20 between (x_k, y_k) from stage k and the optimal solution. Instead, they are bounded by the distance
21 from (x_k, y_k) to the corresponding optimal solutions to the minimization and maximization defined at
22 (x_{k+1}, y_{k+1}) , i.e., $\|x_k - \hat{x}_R(y_{k+1})\|$ and $\|y_k - \hat{y}_R(x_{k+1})\|$ (cf. the key Lemma 3). More importantly,
23 we have to show that this distance is strictly less than the imposed radius R such that adding the
24 bounded ball preserves the duality gap of the original problem. Please note that such interior-point
25 argument is very important and is not necessary in Hazan & Kale.

26 **Q3:** Why avoiding deterministic updates as in [36,32]?
27 A: The reason is two-fold: (i) the deterministic updates in [36, 32] require a specific form of objective
28 function; hence by avoiding deterministic update we are able to handle more general problems without
29 scarifying the complexity; (ii) the deterministic updates in [36, 32] have additional computational
30 overhead, which usually needs to pass all data in machine learning applications. A key difference
31 from [36, 32] is that we use the recursion on the duality gap as for convergence analysis, while
32 [36,32] use the recursion on the primal objective gap for convergence analysis.

33 **Reviewer 2: Q1:** Key technical contribution (Lemma 1) is simple to prove, so unclear why it is
34 important to extend Epoch-GD for SC min problem to Epoch-GDA for SCSC min-max problem.
35 A: We agree Lemma 1 is simple to prove. But **the key for proving the fast rate of duality gap for**
36 **SCSC problems lies at Lemma 3**, which proves that the duality gap of the problem defined with
37 the ball constraint is equal to the original duality gap. This proof is subtly different from that of
38 Epoch-GD. Please refer to response to Q2 of reviewer 1.

39 **Reviewer 3:** Thanks for pointing out the relevant reference. We will add it in the revision.
40 **Q1:** Why we always have $\text{dist}(0, \partial P(\hat{x}_\tau^*)) \leq \gamma \|\hat{x}_\tau^* - x_0^*\|$ in Theorem 2?
41 A: Thanks for pointing this out. Indeed, we should use $\hat{P} = P + \mathbb{I}_X$ in place of P in the above
42 inequality, where \mathbb{I}_X is the indicator function of the set X , which gives us the desired result. To
43 prove this, let us consider an unconstrained ρ -weakly convex function $\psi(x)$ and a reference point
44 \tilde{x} , $f(x) = \psi(x) + \frac{\gamma}{2}\|x - \tilde{x}\|^2$ is $(\gamma - \rho)$ -strongly convex. Hence, we have the unique optimal
45 solution of $\min_x f(x)$, say \hat{x} , then the optimality condition gives that $0 \in \partial\psi(\hat{x}) + \gamma(\hat{x} - \tilde{x})$, i.e.,
46 $\gamma(\tilde{x} - \hat{x}) \in \partial\psi(\hat{x})$, which means $\text{dist}(0, \partial\psi(\hat{x})) \leq \gamma\|\hat{x} - \tilde{x}\|$. Applying this argument to $P(x) + \mathbb{I}_X$
47 leads to the corrected inequality.

48 **Q2:** Theorem 1 provides a high probability result, while Theorem 2 proves a bound in expectation?
49 A: Thanks for noticing this difference. We prove the expectation result for WCSC in Theorem 2 for
50 consistency with previous results [32]. Indeed, we followed your suggestion and found that Thm.
51 2 can be also extended to high-probability result. The key idea is similar to that for proving Thm.
52 1. In particular, we can prove a high-probability result of Lemma 4 similar to Lemma 2. Then by
53 appropriately setting the radius R_k according to η_k and T_k we can able to prove a similar result as in
54 Lemma 3, which leads to a high-probability upper bound for the duality gap of $f_k(x, y)$. From this
55 point, we can prove the high-prob convergence for the WCSC similar to the existing proof of Thm. 2
56 except replacing expectation result with high-probability result. We will discuss this in the revision.