

1 We thank the reviewers for valuable and thoughtful feedback, and for acknowledging the importance of this work.
2 **Scope:** Our main objective has been to provide insights on current limitations of the Neural ODE framework, and to
3 design novel solutions backed by theory. Although evaluations for specific use cases e.g. CNF was not in within our
4 primary scope, we include additional results to address requests and provide further evidence of practical usefulness.
5 **Further augmentation evaluations [R2, R3]:** We obtained improved results on all variants by means of a similar
6 architecture equipped with a pooling layer (closer to standard image class. approaches) and tolerances 10^4 . The neural
7 ODEs learn richer dynamics (higher NFEs), boosting performance across all models and clarifying the relative ranking
8 of aug. approaches. This also leads to improved parameter efficiency ($\approx 8x$ less parameters). Tab. 1 includes results
9 on both MNIST as well as CIFAR10 (R1). The parameter efficiency of 2nd-order models (R2, R3) is now more
10 pronounced. We also report that they converge faster, often several epochs ahead of the alternatives.
11 **Discussion on CNFs [R1, R2, R4]:** We further showcase *data-control* (DC) strategies in the context CNFs as a more
12 complex task. Compared to regular CNFs, DC-CNFs do not require changes to the formulation and converge faster
13 with simpler flows as shown in Fig. 1, effectively reducing NFEs. Adaptive-depth models are also compatible with
14 CNFs and would allow the model to allocate more *depth* to data further away from their target destination.
15 **Depth-variance techniques [R2]:** We agree that the choice of basis in Galérkin neural ODEs is important and worthy
16 of several standalone investigations. However, regarding the *sinusoids* example, periodicity of the weights (inferred by
17 the choice of the Fourier eigenbasis) does not imply periodicity of the Neural ODE and hence does not constitute a
18 strong inductive bias. To confirm this, we tested with different signals and eigenbasis (Chebychev poly., RBFs). Fig. 2
19 shows a more complex experiment for time-varying nonlinear system.
20 **Signal tracking [R2]:** Here, depth-variance is not needed to actually learn the trajectory, generated by $\mathbf{x} =$
21 $x; [x_0; \dot{x}_0] = [1; 0]$ which does not contain any depth-varying harmonics. Rather, it ensures that for any initial
22 condition of the neural ODE, sampled from $\mathcal{N}([1; 0]; \cdot)$, the solution converges to the signal. The same result can be
23 obtained for nonlinear systems whose solution does not admit a finite spectral decomposition as shown above.
24 **Related work [R3]:** We agree that these important references belong to Section 6 and have made the suggested changes.
25 The approach of latent neural SDEs (and ODEs) is different compared to data-control, which does not require variational
26 inference. It is correct to state that, however, both approaches condition the vector field on data.
27 **Relation to PMP [R3]:** We agree that Th.m 1 is directly derived via classic optimal control theory (PMP) and should
28 be more appropriately referred to as "Proposition". However, including it was necessary for two main reasons: to extend
29 vanilla adjoints to integral loss functions (used in practice for CNFs or signal tracking but not yet implemented) and to
30 set the stage for Th.m 2. We note that Th.m 2 is a non-trivial generalization to infinite dimensional spaces.
31 **Relation to COD [R3]:** We agree on COD and clarified the statements in Sec. 5. The phenomenon we want to highlight
32 is that dimension of the state-space also drastically affects the behavior of dynamical systems in general.
33 **Guidelines on choosing correct variants [R3]:** In general, we observe data-control to be beneficial in all settings. We
34 agree that additional guidelines on model choice could be useful to the reader; we will add more information.
35 **Given Sec. 5.2, state-space crossing might be possible if each traj. could travel for different amounts of time.**
36 [R2]: In 5.2 we argue that adaptive-depth models can learn the reflection map **without** crossing flows (as they still
37 cannot cross), consistently to what is stated in rest of Sec. 5. This is in fact the main *leitmotiv* of adaptive-depth models.
38 **Clarifications: Figure 1 [R2]:** The blue curves are learned flows of test init. cond., which converge to the signal
39 to track (hence the decreasing variance across *depth*). **Figure 2 [R2, R3]:** Each traj. represents the evolution of a
40 single parameter. **Training details [R2]** On the signal tracking task, we train on 10^2 initial conditions, full batch. The
41 GalNODE architecture has a hidden layer of 64. On depth-varying classification, the architectures have two hidden
42 layers of 32 with a dataset of 10^4 points (dense, to approx. *connected annuli*). **Why s instead of t ? [R1]:** We chose s
43 against t as a more general formulation for the (continuous) depth, to avoid confusion in static settings or whenever
44 time is not directly related to depth-propagation dimension. **"This approach" refers to hypernetworks? [R2]:** Yes.
45 **Typos details [R1, R2, R3] :** We addressed all the remaining typos.

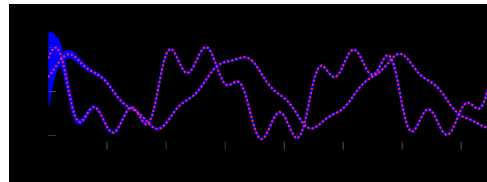
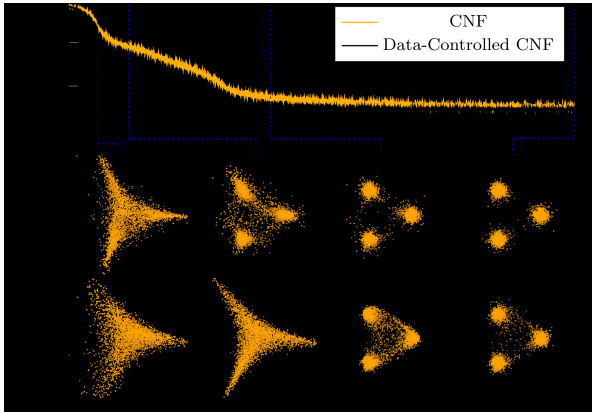


Figure 2: Galérkin Neural ODEs with Chebychev eigenbasis trained with integral loss to track the nonlinear time-varying *Duffing oscillator* $\dot{x} = -\alpha x(1 + x^2) + \beta \cos(\omega s)$ ($\alpha = .5, \beta = 3, \omega = 5$). All traj.s, starting from rand. ICs, converge to the desired signal.

	NODE		ANODE		IL-NODE		2nd-Ord.	
	MNIST	CIFAR	MNIST	CIFAR	MNIST	CIFAR	MNIST	CIFAR
Test Acc.	98.3	59.1	99.1	68.7	99.4	70.7	99.5	71.8
NFE	130	152	124	153	112	150	106	142
Param.[K]	6.4	42.1	6.4	41.4	6.4	41.9	5.8	37.4

model converges faster to a better solution and with simpler flows (thus lower NFEs). Table 1: Test results, 5 runs on MNIST, 3 on CIFAR10² (final results might vary slightly).