Author response for NeurIPS submission 4700 (Latent distance estimation for random geometric graphs)

We are very grateful to all three reviewers for their time, valuable feedback and suggestions. We highly appreciate the encouraging comments regarding the novelty and solid mathematical analysis of our approach.

I. Motivations and related work. We agree with the reviewers that more motivation on the spherical setting would strengthen our paper. The model on the sphere has received attention lately, see for example [1] and references therein. One of our contributions is to point out that the spectrum of these graphs is highly structured, which it may have been unnoticed, and to give a method to recover the distances based on this fact. Also, our work may serve to identify the presence of a geometric representation (spherical) by looking at the spectrum of the graph. In terms of modelling, as noted in [1] the sphere would be an appropriate embedding space when each coordinate (feature) of a given point have the same importance in the determination of the geometric representation.

Reviewer 3 raised the question of the RDPG model. In general, RDPG model considers latent points $\{X_i\}_{i=1}^n$ and the connection probability is a scaled version of $\langle X_i, X_j \rangle$. In our setting, it corresponds to the link function $f(t) = \frac{1}{2}(1+t)$.

II. Analisys. Reviewer 1 pointed out that the event $\mathcal{E}$ "holding ‘for n large enough" may "seem week". One can derive an explicit bound on n using equation (1) in Sec. 3.1 of the supplementary material. We get that is sufficient that:

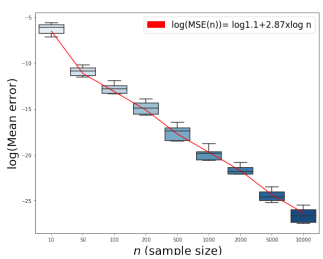
$$\max\{\sqrt{\frac{\rho_n}{n}}, \frac{\sqrt{\log n}}{n}\} \leq \frac{{\Delta^*}^2}{2^{15/2} C\sqrt{d}} \quad \text{and} \quad \frac{\log n}{n} \leq \big(\frac{\Delta^*}{8C'}\big)^{\frac{2s+d-1}{s}}$$

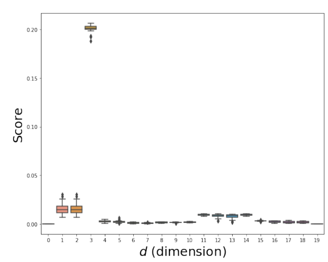
where $C, C' > 0$. We agree that this would shed light on the relation between n, ρ_n and the parameters Δ^*, d and s.

As Rev. 1 mentions one of our contribution is the adaption of matrix perturbation results to a “nearly" orthogonal case, which is detailed in Sec.3 of the supplementary material. Also, it is correct that the Sobolev rate comes mainly by spectral approximation of T_W by T_n. We agree that to explicit both points on the main paper will be useful.

III.Experiments. As suggested by Rev. 1, we include the boxplot for MSE_n accompanied with a curve of the form $MSE_n = Cn^{-r}$ where r is the rate. Here we have a rate $r = 2.87$ for $MSE_n = \frac{1}{n^2}\|\hat{\mathcal{G}} - \mathcal{G}^*\|_F^2$.

Rev. 3 asks about an intuitive explanation for the local maxima in the score function, in the dimension recovery method. Given that $d = 3$ the eigenvalue multiplicities are $1, 3, 5, 7, \cdots, 2k+1, \cdots$ for $k \in \mathbb{N}$, thus is not forbidden that the score peaks at any of those values or at a sum of them (meaning that the corresponding eigenvalues are very close). Also, we found a typo in our code and redo the score boxplot for $n = 2000$. The first two figures will replace Fig. 1 of the main paper. In addition, we include the mean (25 rep.) runtime of HEiC alg. for different values of n and correct the typo in the HEiC alg. description. Given that HEiC is spectral algorithm, it will scale roughly as n^3.





n(sample size)	runtime(seconds)
10	0.012
50	0.016
100	0.020
200	0.040
500	0.19
1000	1.02
2000	5.07
5000	62.37
10000	450.53

P.C specs: 3,3GHz Intel i5, 16 GB RAM

Algorithm 1 Harmonic EigenCluster(HEiC) algorithm

Input: $(\hat{T}_n, d)$ adjacency matrix and sphere dimension
$\Lambda^{\text{sort}} = \{\hat{\lambda}_1^{\text{sort}}, \cdots, \hat{\lambda}_{n-1}^{\text{sort}}\}$ ←eigenvalues of $\hat{T}_n$ sorted in decreasing order
$\Lambda_1 \leftarrow \{\Lambda_1^{\text{sort}}, \cdots, \Lambda_{1+d}^{\text{sort}}\}$: where Λ_i^{sort} is the i-th element in Λ^{sort}
Initialize $i = 2$, $gap = Gap_1(\hat{T}_n; 1, 2, \cdots, d)$
while $i \leq n - d$ **do**
  if $Gap_1(\hat{T}_n; i, i+1, \cdots, i+d) > gap$ **then**
    $\Lambda_1 \leftarrow \{\Lambda_i^{\text{sort}}, \cdots, \Lambda_{i+d}^{\text{sort}}\}$
  end if
  $i = i + 1$
end while
Return: Λ_1, gap

IV. Extensions and future work. As Rev. 2 points out, the spherical case can serve as a building block towards more complex models. An ongoing work of the authors is the extension to the Euclidean unit ball where nodes closer to the border will be more connected than the nodes closer to the center, allowing for more interesting applications. We agree with Rev. 3 that graphex models will be worth exploring to extend our method to the sparse case.

References

[1] S. Bubeck, J. Ding, R. Eldan, and M. Rácz. Testing for high dimensional geometry in random graphs. *Random Structures and Algorithms*, 49:503–532, 2016.