<sup>1</sup> We thank all the reviewers for their detailed feedback.

## 2 Reviewer 3:

<sup>3</sup> We believe the proof of our lower bound is correct as written. In our lower bound, the rewards  $r_{i,t}(c_j)$  do have an

4 explicit dependence on  $c_j$ ; specifically we write that "assign rewards so that  $r_{i,t}(c) = 0$  if t is in the *j*th epoch and

5  $c \neq c_j$ " (otherwise we assign rewards according to some hard bandit instance). The dependence on  $c_j$  comes from the 6  $c \neq c_j$  clause. In other words, during the *j*th epoch in which the context is  $c_j$ , the rewards of all other contexts are zero.

 $c \neq c_j$  clause. In other words, during the *j*th epoch in which the context is  $c_j$ , the rewards of an other contexts are zero.

7 We do not understand what the reviewer means by "extending the existing proof of EXP3" to the contextual cross-

<sup>8</sup> learning setting. We certainly use the general proof structure of EXP3 (as do very many learning algorithms in settings

<sup>9</sup> with adversarial rewards), but the core technical contribution here is figuring out how to extend the EXP3 algorithm to <sup>10</sup> the contextual setting with cross-learning. The most direct such approach (updating all arms/contexts you have access

the contextual setting with cross-learning. The most direct such approach (updating all arms/contexts you have access to the exact same way as in EXP3) results in our EXP3.CL-U algorithm, which results in a suboptimal  $\tilde{O}(K^{1/3}T^{2/3})$ 

to the exact same way as in EXP3) results in our EXP3.CL-U algorithm, which results in a suboptimal  $O(K^{1/3}T^{2/3})$ regret by the standard EXP3 analysis. Extending this to EXP3.CL involves constructing a low-variance unbiased

13 estimator via importance sampling, which we believe is a novel contribution in this setting.

<sup>14</sup> We believe that the proof argument in Section 4 of the "Bandits with concave rewards and concave knapsacks" paper

<sup>15</sup> can be adapted to the contextual setting to recover an  $O(\sqrt{KT})$  regret bound for UCB1.CL (we thank the reviewer

<sup>16</sup> for pointing out this alternate proof). However, to the best of our knowledge, this technique 1. does not allow us to

prove gap-dependent bounds that have logarithmic dependency in T and 2. does not extend to the partial cross-learning setting (here the inequality in (2) breaks). Since our technique in the proof of Theorem 1 nicely generalizes to these

settings, we believe it has technical merit. We plan to mention this in the updated version of the paper.

## 20 Reviewer 4:

We thank the reviewer for suggesting an experimental comparison with LinUCB; we agree it is an interesting direction for future work.

## 23 Reviewer 5:

We understand that the assumption that the learner's value is independent from the other bidder's bids is a strong assumption. We have the following thoughts about this subject:

- If we condition on enough public features of the query, this can significantly decrease the correlation between different bids (of course, this leads to an interesting question of how to cross-learn between these new contexts given by public features).
- In particular, many advertisers in display advertising markets base bids on cookies, which are information stored on users' browsers. Because cookies are private, cookie-based bids are typically weakly correlated. For example, Amazon knows that a user is interested in purchasing shoes because she searched for shoes on its website, but a competitor such as Macy's might not have this information.
- In our experiments, the data does not have the property that the values (contexts) are independent of the other
  bids (in fact, the linear correlation between the two is sizeable). Nonetheless, the cross-learning algorithms
  designed in the paper seem to perform well, and understanding this robustness is an interesting direction for
  future work.
- Part of the motivation for the development of contextual bandits with partial cross-learning is exactly to address this issue; even if correlation is present, you might have some idea of which other contexts it is safe to cross-learn over.

<sup>40</sup> The reviewer asked where we obtain the values for the bidder in our simulation. This bidder directly provided their true

valuation for each query (specifically, what they would have bid for the same query in a truthful second price-auction)

to us for the purpose of this research project. We will clarify this in the updated version of the paper.