

1 We thank all the reviewers for their thorough reviews and insightful comments. Reviewers’ comments are in blue.

2 **Reviewer 1:** *The authors should explain more how this is actually done, and why it doesn’t represent a computational*  
3 *bottleneck for the proposal.* The detailed algorithms for finding robust error regions under  $\ell_\infty$  and  $\ell_2$  perturbations  
4 are provided in Section C in our supplementary materials. For  $\ell_\infty$ , we construct the systems of hyperrectangles by  
5 first precomputing an approximate k-NN distance estimate using Ball Trees for each data point, and then clustering  
6 the top- $q$  densest data points into  $T$  partitions using the k-means algorithm, where we binary search for the optimal  
7 parameter  $q$ . The time complexity of precomputing and sorting the nearest neighbor distance estimates is approximately  
8  $O(nd \log(n))$ , where  $n$  is the total number of data points in  $\mathbb{R}^d$ . In addition, the time complexity of k-means algorithm  
9 is  $O(ndTI)$ , where  $I$  is the averaged number of iterations for k-means algorithm to converge. Therefore, the total  
10 computational complexity of our algorithm for  $\ell_\infty$  is  $O(nd \log(n) + ndTI \log(1/\delta))$ , where  $\delta$  is the stopping threshold  
11 for binary search. In our experiments, we applied our algorithms to medium-sized datasets including CIFAR-10 and  
12 SVHN, and they finished reasonably quickly. We will include a runtime analysis of algorithms in the final version.

13 **Reviewer 2:** *It is worth including a discussion on <https://arxiv.org/abs/1805.12152>, which many researchers point to*  
14 *as refuting the concentration of measure hypothesis.* We will include the discussions. In a nutshell, that work uses a  
15 definition that coincides with the definition of adversarial examples that we use for the interesting range of tampering  
16 parameters (in which the ground truth is robust). But, if the tampering goes up and can change the ground truth, even  
17 learning a concept *exactly* might leave room for adversarial examples under the definition used in that work (but not  
18 under ours). *I would also be a bit more specific in the introduction that  $l_p$  perturbations are a toy threat model that is*  
19 *not intended to model how actual adversaries choose to break systems.* We will make sure to add comments about  
20 shortcomings of  $l_p$  norm in capturing the whole picture. *Any reason the authors defined an adversarial example as the*  
21 *nearest input which is classified differently, and not the nearest error?* We agree with you that nearest error point is a  
22 natural definition of adversarial examples and we indeed use this definition in the paper. Specifically, in Definition 2.1,  
23 we compare the true label of  $x'$  with the predicted label of  $x'$ , which means  $x'$  should be an error point to be counted as  
24 an adversarial example. We will make this point more clear in the statement of our definition. *It’s worth noting that*  
25 *MNIST may be a degenerate case with respect to the  $l_\infty$  metric. In particular, a trivial defense is to first threshold the*  
26 *inputs about .5 and classify the resulting binary image. Because of this, I would not expect any meaningful bounds*  
27 *to hold for this dataset and metric.* We agree with the reviewer that thresholding MNIST (and any other dataset) will  
28 make the transformed distribution not concentrated under  $l_\infty$ . However, the original distribution (before transformation)  
29 might still be concentrated. In particular, one might be able to add a small perturbation to the image before thresholding  
30 the features and make the binary transformation of the perturbed image different from that of the original image. For  
31 the case of MNIST, it seems that binarizing images should not change the distribution much, as the original images  
32 have close to binary form. Our experiments support this intuition and show that regions in MNIST dataset could have a  
33 very small expansion (it only grows from  $\sim 1\%$  to  $\sim 10\%$  when allowing  $\epsilon = 0.4$  perturbations). *It would be very*  
34 *interesting if the authors could strengthen their bounds by making additional assumptions on the shape of the error set.*  
35 *Additionally, one could strengthen the bounds by approximating the content-preserving threat model.* Thank you for  
36 pointing out these interesting future directions, particularly for the content-preserving mode. Interestingly, part of our  
37 theoretical results do already prove such results for restricted forms of error sets, and this does set the stage on how we  
38 choose the sets for our experiments. The *proof* of Theorem 3.5 first proves such result for limited shapes. In particular,  
39 we obtain such result when VC dimension of the sets *and* their expansion are bounded (e.g., union of hyperrectangles).

40 **Reviewer 3:** *Q1- the theoretical innovations in this paper are not practically relevant to the study of adversarial*  
41 *vulnerability. Q2- to disprove the "adversarial examples are inevitable" theory, you only need to show an \*upper bound\**  
42 *on the concentration function, i.e. a finding that there exists some set with measure alpha whose epsilon-expansion*  
43 *has measure at most Y. Given a sample from the data distribution, here is a simple way to do that: split the sample*  
44 *into a "training set" and a "test set".* The two questions/comments are relevant. We start with Q2, then will address  
45 Q1 as well. Yes, indeed to show that a distribution does not concentrate beyond a parameter, one can aim to show the  
46 existence of *some* set (found based on “training set”) and test its expansion using the “test set”. However, the question  
47 is how to design algorithms that come up with such sets. Our theory tells us that by looking at specific types of sets  
48 (e.g., collection of hyperrectangles), we can get “generalization” bounds for estimating expansion. Note that we tried  
49 different collections of subsets (e.g., subsets decided by neural networks) that were not supported by our theory and we  
50 observed huge generalization error that made the experiment meaningless. Therefore, in our experiments we use exactly  
51 the subset collections that theory suggests and the results of experiments verify our theory. Our theory is also important  
52 for *future* work. If one wants to find the concentration of measure under another metric probability space, they can  
53 use our theory to come up with suitable subset collections with generalization guarantees. *As a sidenote, while the*  
54 *authors interpret this to mean that there is room to develop better robust classifiers, it could also mean that robustness*  
55 *is impossible for reasons other than concentration of measure.* Thank you for pointing this out. We tried to be cautious  
56 in interpreting our results and consider the “robust classification is impossible for other reasons” hypothesis. However,  
57 after reading through the paper we found an occasion in our discussions (line 284-285) that we did not consider this  
58 hypothesis. We will make sure to clarify this in the next version of our paper.