

1 **R1:**

- 2 • **Motivation and context:** the proposed estimator is motivated with M/EEG applications in mind. For the sake of  
3 clarity, the M/EEG part of the introduction has been rewritten thanks to the insightful comments. We now detail that  
4 in the M/EEG setting the noise is correlated between sensors (non-diagonal covariance), but also that three types of  
5 sensors are potentially available (gradiometers, magnetometers and electrodes). We now also clarified that 3 types  
6 of sensors can be used to increase the number of measurements (samples), yet each sensor type measures different  
7 physical quantities and have different noise characteristics (heteroscedasticity). To avoid any ambiguity we will  
8 replace the word ‘heteroscedastic’ by ‘correlated’ in the title. MTL has been recalled in the introduction to make the  
9 connection more explicit.
- 10 • **Repetitions in the M/EEG context:** the repetitions concern the cognitive experiment (eg, recording M/EEG signal  
11 for 1 s following an auditory stimulation on one patient). The same experiment is performed, typically 50 times,  
12 sequentially on the same patient, which results in 50 repetitions of all sensors measurements. Alternative word for  
13 repetition in this context is *trial*.
- 14 • **Hyperparameter tuning** is a difficulty shared by all the compared methods. Popular approaches include (gen-  
15 eralized) cross-validation. We wanted to decouple the two possible causes of errors: the one due to imperfect  
16 hyperparameter setting and the one due to the estimator itself. That’s why **1)** on synthetic and realistic data we  
17 provided support recovery ROC curves (with a wide range of  $\lambda$ s) **2)** on real data we fixed the number of non-zero  
18 coefficients to 2 (for this dataset using auditory stimuli one expects one active source in each auditory cortex), and  
19 we selected a corresponding  $\lambda$ . It allowed us to compare methods performance irrespective of the selection of the  
20 regularization parameter.
- 21 • **Notation:** when the norm subscript is a real number  $p$ ,  $\|\cdot\|_p$  refers to the classical  $\ell_p$  norm (L66), and for a (positive  
22 definite) matrix  $S^{-1}$ , it refers to the associated Mahalanobis norm (L71). We will detail this subtlety in the notation.
- 23 • **“why does the homoscedastic solver fail after whitening? (L222)”:** we may have been unclear. The sentence  
24 L222 is “**without** this whitening process, the homoscedastic solver MTL fails”. Indeed MTL is the  $\ell_{2,1}$  regularized  
25 Maximum Likelihood Estimator with an iid (white) Gaussian noise modelling assumption. We meant that when this  
26 iid assumption breaks, e.g. when data are not whitened, MTL fails. We will rephrase to avoid any confusion.

27 **R2: Overlap with Massias 2018a** Our work builds actually on Massias 2018a. Whereas this paper focused on how to  
28 solve optimization problems like SGCL (Eq. 4), our main theoretical contribution is to show how SGCL and CLaR  
29 result from the smoothing theory of Nesterov applied to a matrix valued function. As pointed out by R1: “*the authors*  
30 *provide a theoretical justification for their approach*“ while the formulation introduced in Massias 2018a was **heuristic**.

31 **Theoretically** we stand from Massias 2018a by providing new contributions:

- 32 • **1-** We provide an **explicit variational formula** for the (smoothed) nuclear norm (see Proposition 4). To be more  
33 explicit: when  $ZZ^T \succ 0$ ,  $\|Z\|_{\mathcal{S},1} = \min_{S \in \mathcal{S}_{++}^n} \frac{1}{2} \|Z\|_{S^{-1}}^2 + \frac{1}{2} \text{Tr}(S)$  (see van de Geer 2016, Lemma 3.4, p. 37).  
34 When  $ZZ^T \not\succeq 0$ , one can approximate  $\|Z\|_{\mathcal{S},1}$  by the following formula:  $\|Z\|_{\mathcal{S},1} \approx \min_{S \in \mathcal{S}_{++}^n} \frac{1}{2} \|Z\|_{S^{-1}}^2 +$   
35  $\frac{1}{2} \text{Tr}(S) + \frac{1}{2} (\underline{\sigma}/2)^2 \text{Tr}(S^{-1}) = \sum_i \sqrt{\gamma_i^2 + (\underline{\sigma}/2)^2}$  (see Optimization with Sparsity-Inducing Penalties, Bach et  
36 al. 2012, p. 62), which is a  $\underline{\sigma}/2$ -smooth  $\underline{\sigma}(n \wedge q)/2$ -approximation of  $\sum_i \gamma_i = \|Z\|_{\mathcal{S},1}$  (see Beck and Teboulle  
37 2012, Example 4.6, p.573). Here, we proposed a different smoothing ( $\|\cdot\|_{\mathcal{S},1} \square \omega_{\underline{\sigma}}$ ) for which we provide an  
38 **explicit formula:**  $(\|\cdot\|_{\mathcal{S},1} \square \omega_{\underline{\sigma}})(Z) = \min_{S \succeq \underline{\sigma} \text{Id}} \frac{1}{2} \|Z\|_{S^{-1}}^2 + \frac{1}{2} \text{Tr}(S)$  (Prop. 4) which is a  $\underline{\sigma}$ -smooth  $\underline{\sigma}(n \wedge q)/2$ -  
39 approximation (see Beck and Teboulle 2012, Thm. 4.1, p. 567) of  $\|\cdot\|_{\mathcal{S},1}$ . This smooth approximation leads to a  
40 better Lipschitz constant for a given  $\varepsilon$ -approximation. We will highlight such a **theoretical** and practical **benefit**  
41 (faster convergence thanks to a better Lipschitz constant) of our approach.
- 42 • **2-** The proposed smoothing approach (see App. A) paves the way to a practical use of Schatten norms as datafitting  
43 terms by not requiring to solve problems where both datafitting and regularization terms are non-smooth; see in  
44 particular the smoothing of the Schatten 2 (App. A.5) and Schatten  $\infty$  (App. A.6) -norm.

45 **Empirically** we stand from Massias 2018a with the following contributions: **1)** the **modelling** contribution with  
46 the repetitions for M/EEG **2)** the **extensive benchmark** against convex and non-convex estimators (with a clean  
47 opensource package pointed out by R3 “the provided code helped me a lot to digest some technical pieces“) **3)**  
48 **extensive experiments on real data** (Fig.6 and 7 + App. D) with potential impact for the neuroscience community  
49 (R3 “Thanks to the provided code, the impact of the paper could be immediate and more probable“). **4)** We report that  
50 solving CLaR is as computationally cheap as solving SGCL, see App. B.7 and Tab. 1.

51 **R3:**

- 52 • **clarifications about M/EEG context:** (see answer to R1) we will better explain the specificities of the M/EEG  
53 framework in the introduction.
- 54 • **sample** is a publicly available M/EEG dataset included in the Python package MNE. It consists in measurements  
55  $Y^{(1)}, \dots, Y^{(r)}$  corresponding to auditory or visual stimulations. This has been clarified in the paper.