

1 We thank the reviewers for their thoughtful suggestions which we'll incorporate to substantially improve our final paper.

2 **Experiments.** It is not easy to integrate bandit systems without extensive industry resources, particularly in a sensitive
3 area like pricing. Nearly all past research on general bandits/pricing methodologies relied on simulation experiments
4 (many papers just provide regret bounds without experiments). No matter which experiments we run, the sensitive
5 nature of pricing necessitates provable guarantees, which is a major strength of our adversarial regret bounds. We'll
6 emphasize our robustness analysis under misspecified d (Fig C.1) and a totally-misspecified demand-model (Fig C.2).

7 **R1:** Seems odd that the regret can be decreasing in time (which happens for figures C, D, E and F). How is this possible?

8 L267 states: "regret of the bandit algorithms decreases over time, indicating they begin to outperform the optimal
9 fixed price chosen in hindsight". We'll clarify: Since our bandits can vary price over time and these environments are
10 nonstationary, our algorithms are able to outperform *any* single fixed price-configuration (what regret is defined against).
11 This limitation of the standard regret definition has led to alternative dynamic-regret formulations such as those of
12 [RS2013, Z2017], although dynamic pricing literature typically measures regret against a single price as we've done.

13 [RS2013] Rakhlin, Sridharan. "Online Learning with Predictable Sequences" [Z2017] Zhang et al. "Improved Dynamic Regret for Non-degenerate Functions"

14 **R1:** How does FindPrice (from 0 or p_{t-1}) influence regret? GDG/OPOK/OPOL curves suspiciously similar for $d = N$.

15 We'll clarify: Even comparing GDG vs. itself would result in a (small statistically insignificant) visible difference in
16 curves as bandits are internally stochastic (cf. OPOK Step 4). If $d = N$: GDG & OPOK are nearly mathematically
17 equivalent (same regret bound, but their empirical regret is not identical for the aforementioned reason). Minor
18 difference is action-noising step (ie. Step 5 of OPOK): ξ_t is applied in the p -space for GDG and to x for OPOK. Since
19 $d = N \Rightarrow U$ is $N \times N$ and orthogonal, this makes no theoretical difference for rotationally-invariant uniform ξ_t .
20 FindPrice makes no difference here because $d = N \Rightarrow U$ is invertible. When $d = N$: OPOL and OPOK are also
21 nearly equivalent, because \hat{U} is also orthogonal $N \times N$ matrix. We'll clarify that either choice of FindPrice (from 0 or
22 p_{t-1}) obeys our paper's regret bound (we do not find statistically-significant difference in our experiments). Choice
23 should be based on the seller's philosophy (p_{t-1} : less dramatic price changes, 0: more stability around default price=0).

24 **R1:** I suspect chosen parameters in experiments make task too easy (influence of prices p in the demand seems marginal)

25 We'll clarify: If parameters were chosen to make problem too easy, our figures wouldn't depict such a statistically
26 significant difference in different methods' regrets. We chose z, V, λ to encourage three properties underlying real-world
27 demand curves: different products' baseline demands & demand elasticities should be highly diverse (wide range
28 of z), prices should highly influence demands such that price-increases should severely decrease demand and affect
29 demand for the same product more than other products ($\lambda = 10$ reflects this far better than 0 and leads to V having
30 far bigger values than suggested by $N(0, 2)$). The optimal p^* in a stationary environment has $\|p^*\|_2 \approx 8$, whereas p^*
31 would instead lie somewhere near \mathcal{S} -boundary ($\|p^*\|_2 = 20$) if price didn't substantially influence demand. We did
32 initially set our noise variance =1 as suggested, but wanted to explore noisier settings (ie. harder problems) and found
33 methods could handle 10 without noticeable performance degradation. Results from main text look very similar under
34 variance=1, we'll add them to supplement. In existing supplement experiments, we already use variance =1 (see L584).

35 **Relationship with existing work.** We'll clarify: the main aspect of model (1) that is similar to cited work is the
36 assumption of a linear demand/price relationship¹. Existing work on dynamic pricing is unrealistic as it does not
37 consider multiple products & nonstationary demand curves. Our work is novel because it can handle these cases and
38 obtains even superior performance guarantees when an additional low-rank assumption holds. We do not claim the
39 low-rank assumption is justified by existing pricing work, and instead will cite work on e-commerce recommendations,
40 where low-rank product feature decompositions are a standard assumption that practically works [S2017, Z2016].

41 [S2017] Sen et al. "Contextual Bandits with Latent Confounders: An NMF Approach". [Z2016] Zhao et al. "Predictive Collaborative Filtering with Side Information".

42 **Assumptions.** Fig C.1-C.2 show our methods work well even if our assumptions are wrong. We'll include extra
43 experiment on real demand data² for 1340 products sold by Grupo Bimbo over 7 weeks. We form a matrix \mathbf{Q} of the
44 total weekly demands for each product across all stores. The SVD of \mathbf{Q} reveals the following percentages of variation
45 in the observed demands are captured by top k singular vectors: $k = 1 : 97.1\%$, $k = 2 : 99.1\%$, $k = 3 : 99.9\%$, thus
46 suggesting empirical validity of our low-rank assumption on the demand variation.

47 (A4) is not a strong assumption: up to scaling factors, the orthogonality condition on U does not actually really restrict
48 the family of demand curves that can be captured via our low-rank unknown-features model (there is much flexibility
49 by changing V_t). See also Theorem A.2 in the supplement for alternative (more general) assumptions in case of known
50 features. As stated in L60 & Appendix D, C denotes universal constants. We'll clarify C is problem-independent and
51 does not depend on T, d, r (our usage of C is equivalent to big O notation commonly used to present regret bounds).

¹Historical demand data often nicely fit linear relationship, cf. Houthakker and Taylor (1966) or towardsdatascience.com/calculating-price-elasticity-of-demand-statistical-modeling-with-python-6adb2fa7824d

²www.kaggle.com/c/grupo-bimbo-inventory-demand/