

# Supplementary Material – Recycling Privileged Learning and Distribution Matching for Fairness

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## 1 Complete experimental results on evolutionary multi-objective

### 1.1 Visualization of a Pareto frontier

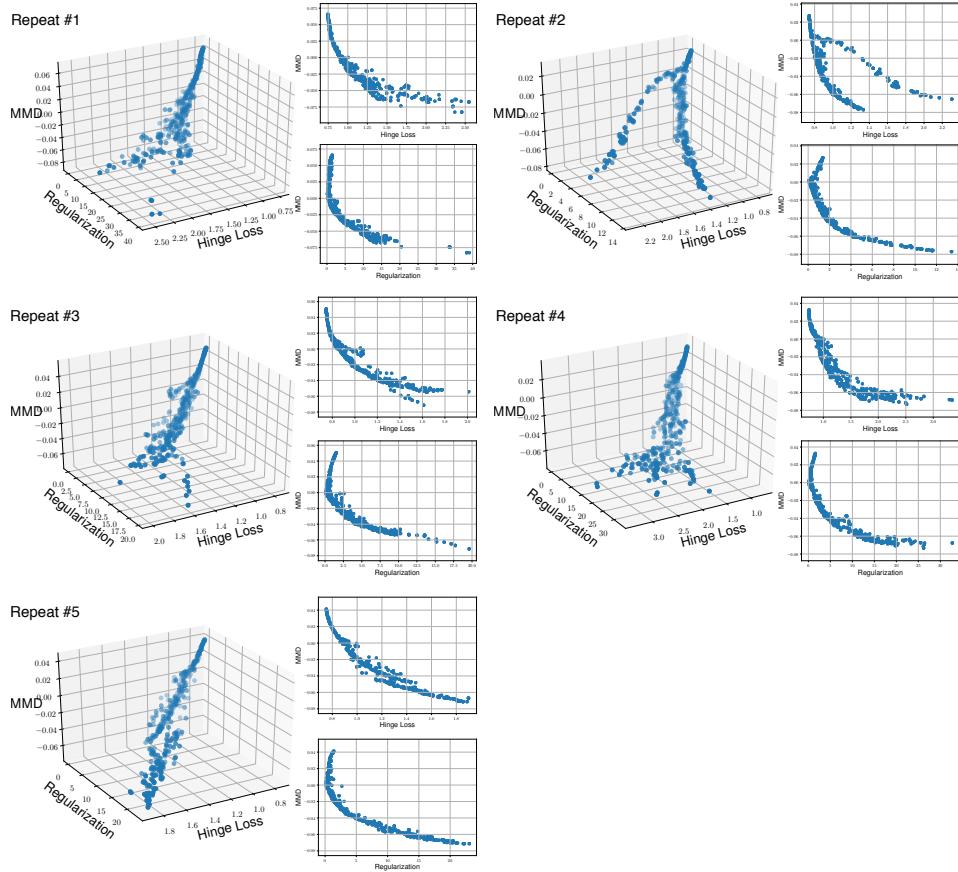


Figure 1: ProPublica COMPAS dataset. Visualization of a Pareto frontier of our DM method. In a 3D criterion space corresponding to the three objective functions: hinge loss, i.e.  $\max(0, 1 - y_n[\langle \mathbf{w}, \mathbf{x}_n \rangle + b])$ , regularization, i.e.  $\|\mathbf{w}\|_{\ell_2}^2$ , and MMD, i.e.  $\text{MMD}(p_{Z=0}, p_{Z=1})$ .

\* Also with National Research University Higher School of Economics, Moscow, Russia.

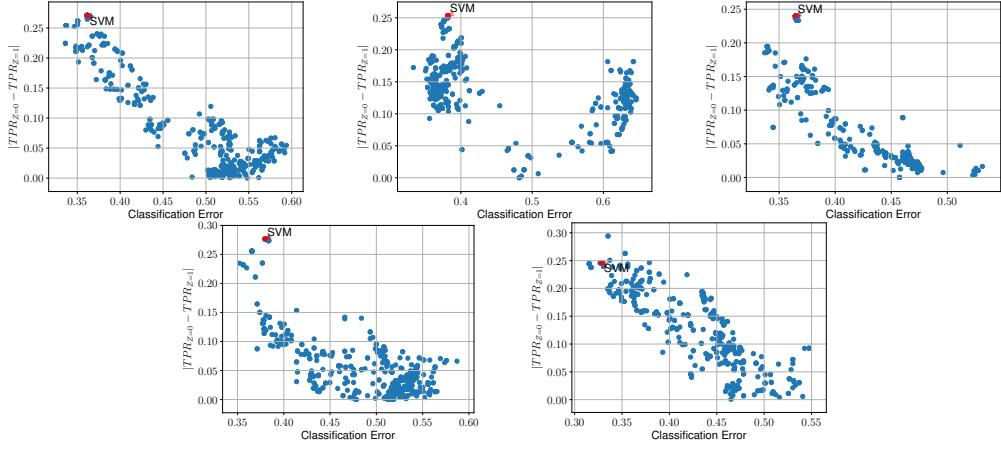


Figure 2: ProPublica COMPAS dataset. Visualization of a Pareto frontier of our DM method. The same Pareto frontier as in Figure 1 but in a 2D space of error and unfairness in predictions.

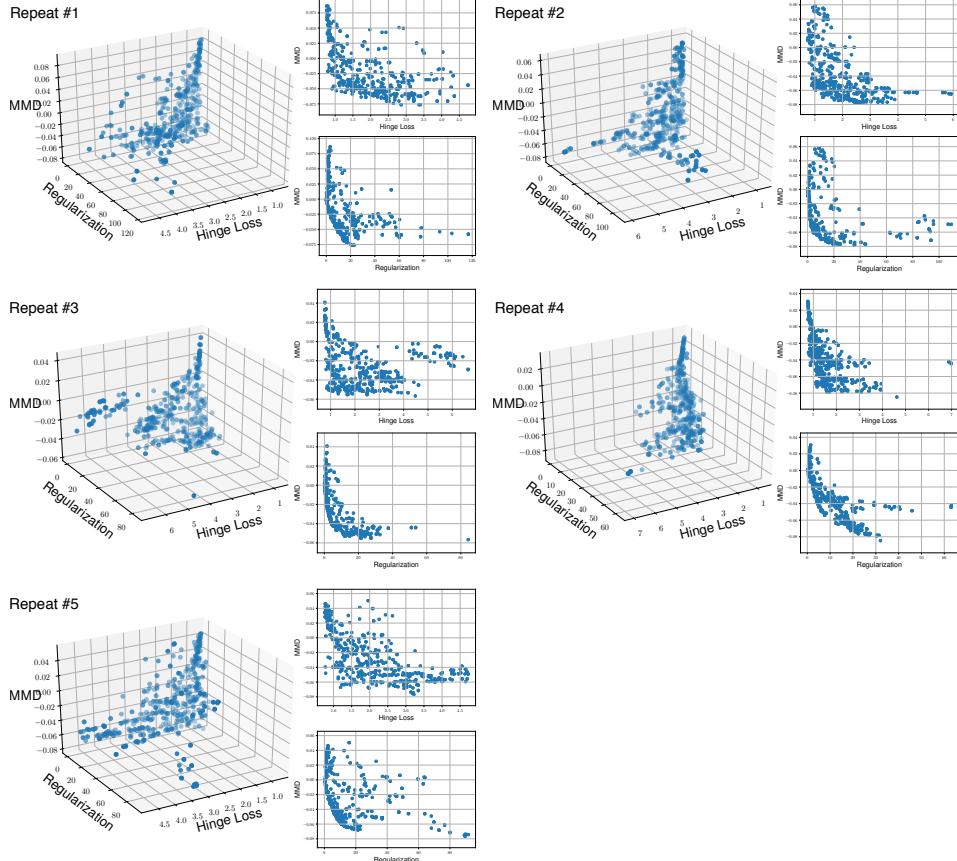


Figure 3: ProPublica COMPAS dataset. Visualization of a Pareto frontier of our DM+ method. For DM+, the criterion space is actually a 5D space, corresponding to the five objective functions: two hinge losses, i.e.  $\max(0, 1 - y_n[\langle \mathbf{w}^*, \mathbf{x}_n^* \rangle + b^*] - y_n[\langle \mathbf{w}, \mathbf{x}_n \rangle + b])$  and  $\max(0, -y_n[\langle \mathbf{w}^*, \mathbf{x}_n^* \rangle + b^*])$ , two regularization terms, i.e.  $\|\mathbf{w}\|_{\ell_2}^2$  and  $\|\mathbf{w}^*\|_{\ell_2}^2$ , and MMD, i.e.  $MMD(p_{Z=0}, p_{Z=1})$ . Only 3D is visualized, corresponding to the MMD, hinge loss  $\max(0, 1 - y_n[\langle \mathbf{w}^*, \mathbf{x}_n^* \rangle + b^*] - y_n[\langle \mathbf{w}, \mathbf{x}_n \rangle + b])$ , and regularisation  $\|\mathbf{w}\|_{\ell_2}^2$ .

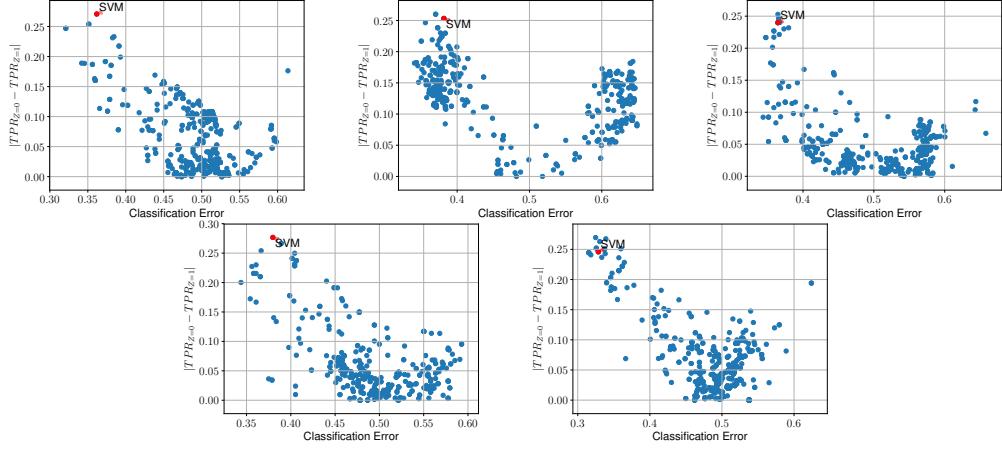


Figure 4: ProPublica COMPAS dataset. Visualization of a Pareto frontier of our DM+ method. The same Pareto frontier as in Figure 3 but in a 2D space of error and unfairness in predictions.

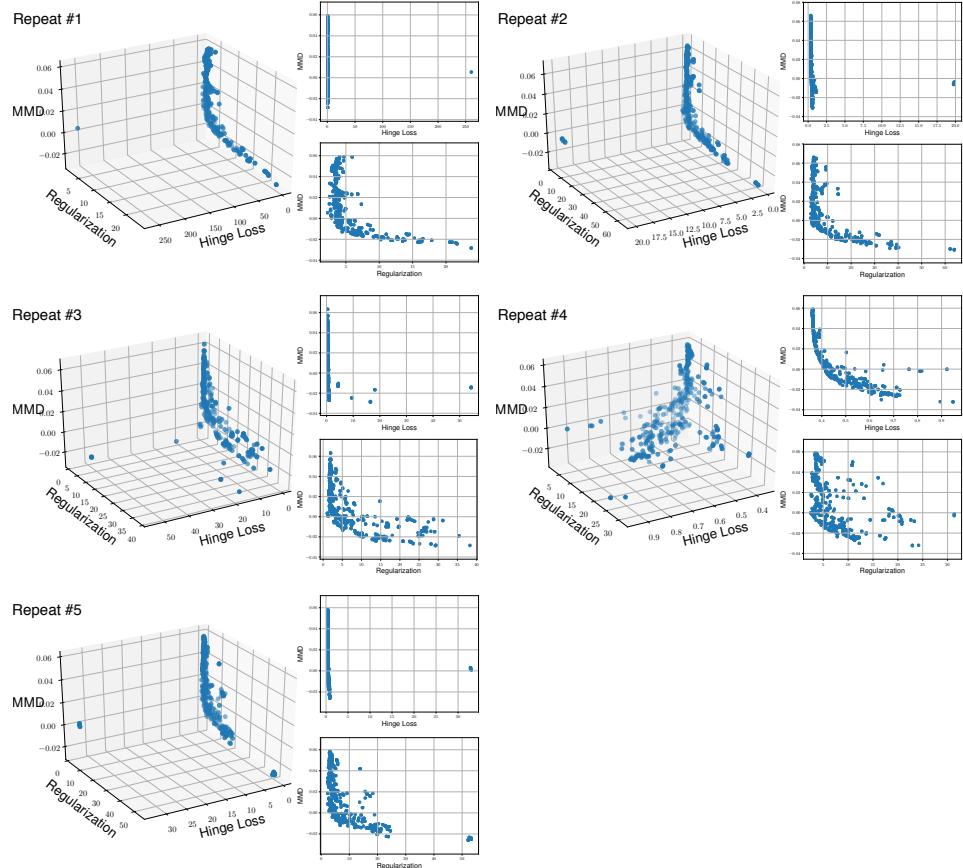


Figure 5: Adult dataset. Visualization of a Pareto frontier of our DM method. In a 3D criterion space corresponding to the three objective functions: hinge loss, i.e.  $\max(0, 1 - y_n[\langle \mathbf{w}, \mathbf{x}_n \rangle + b])$ , regularization, i.e.  $\|\mathbf{w}\|_{\ell_2}^2$ , and MMD, i.e.  $MMD(p_{Z=0}, p_{Z=1})$ .

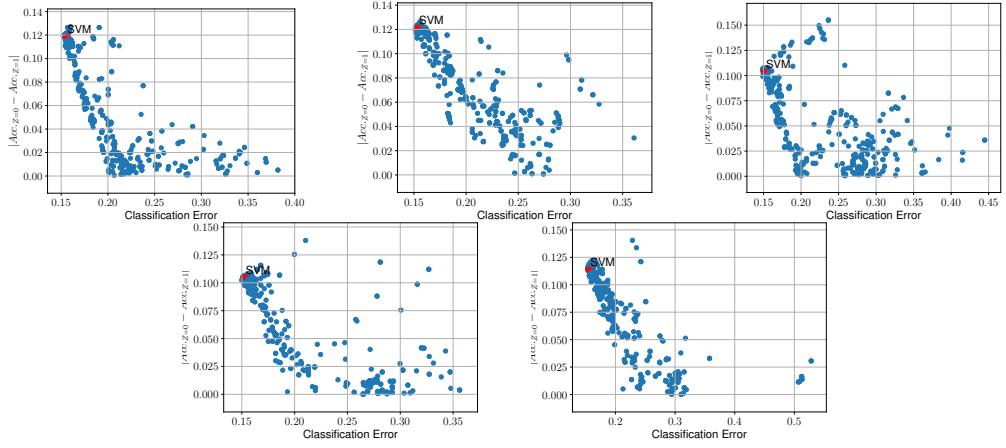


Figure 6: Adult dataset. Visualization of a Pareto frontier of our DM method. The same Pareto frontier as in Figure 5 but in a 2D space of error and unfairness in predictions.

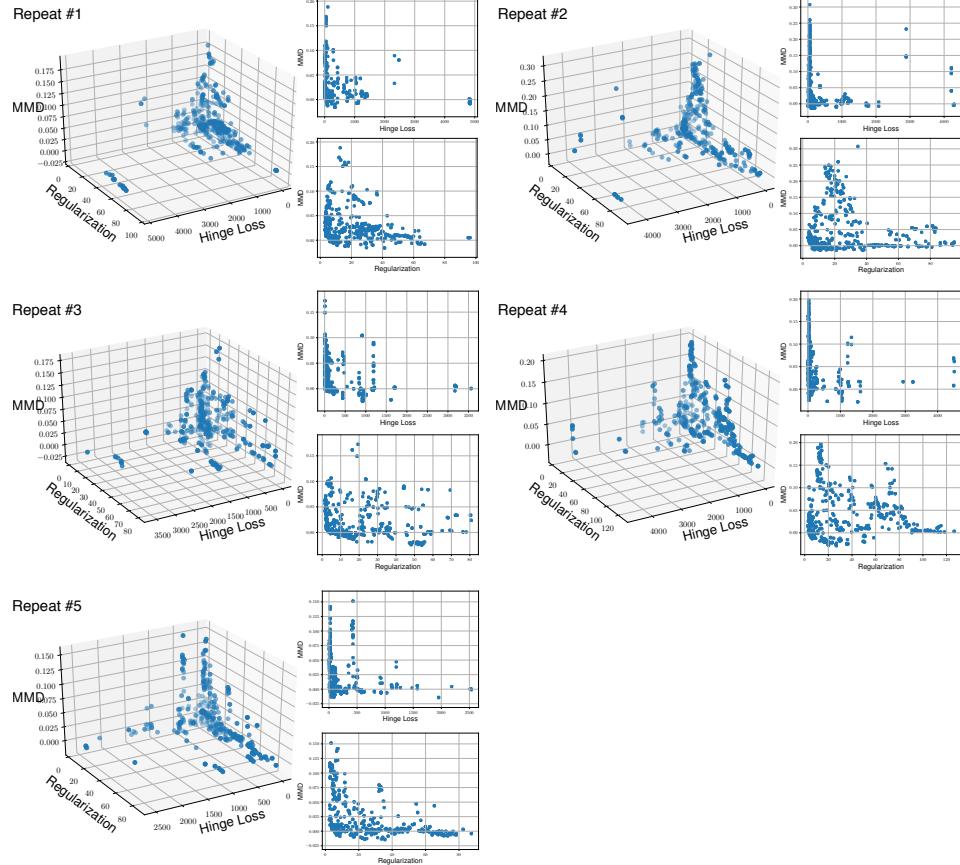


Figure 7: Adult dataset. Visualization of a Pareto frontier of our DM+ method. For DM+, the criterion space is actually a 5D space, corresponding to the **five** objective functions: two hinge losses, i.e.  $\max(0, 1 - y_n[\langle \mathbf{w}^*, \mathbf{x}_n^* \rangle + b^*] - y_n[\langle \mathbf{w}, \mathbf{x}_n \rangle + b])$  and  $\max(0, -y_n[\langle \mathbf{w}^*, \mathbf{x}_n^* \rangle + b^*])$ , two regularization terms, i.e.  $\|\mathbf{w}\|_{\ell_2}^2$  and  $\|\mathbf{w}^*\|_{\ell_2}^2$ , and MMD, i.e.  $MMD(p_{Z=0}, p_{Z=1})$ . Only 3D is visualized, corresponding to the MMD, hinge loss  $\max(0, 1 - y_n[\langle \mathbf{w}^*, \mathbf{x}_n^* \rangle + b^*] - y_n[\langle \mathbf{w}, \mathbf{x}_n \rangle + b])$ , and regularisation  $\|\mathbf{w}\|_{\ell_2}^2$ .

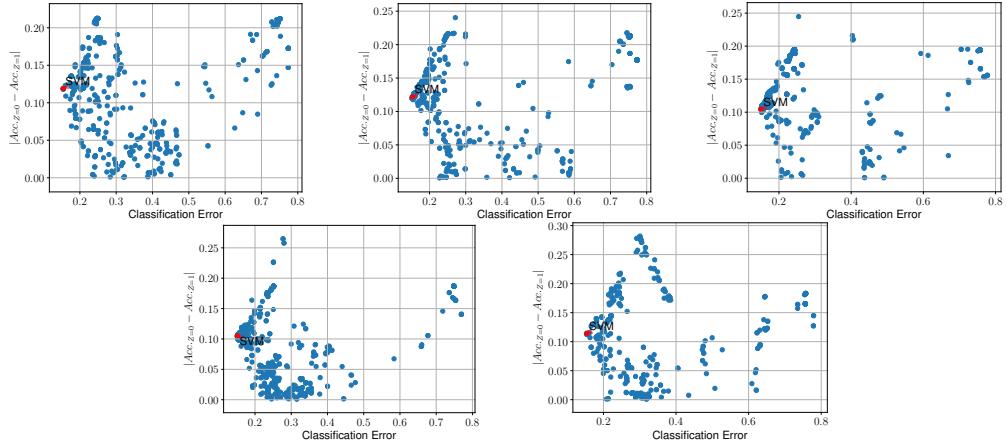


Figure 8: Adult dataset. Visualization of a Pareto frontier of our DM+ method. The same Pareto frontier as in Figure 7 but in a 2D space of error and unfairness in predictions.

## 1.2 Selection of an operating point

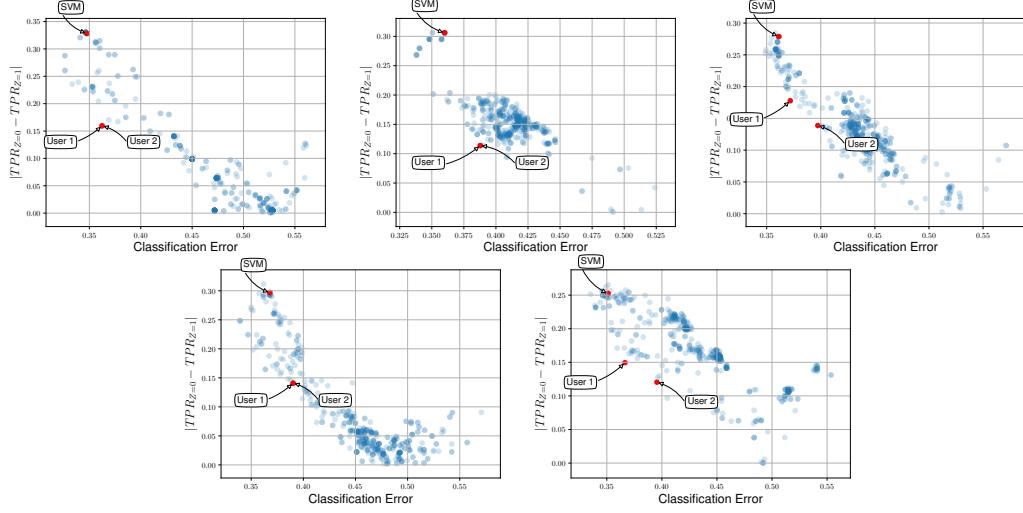


Figure 9: ProPublica COMPAS dataset. The above operating point selection is based on 40% of the training data. User 1 has an inclination to be more lenient in being fair for a gain in accuracy in comparison to the User 2.

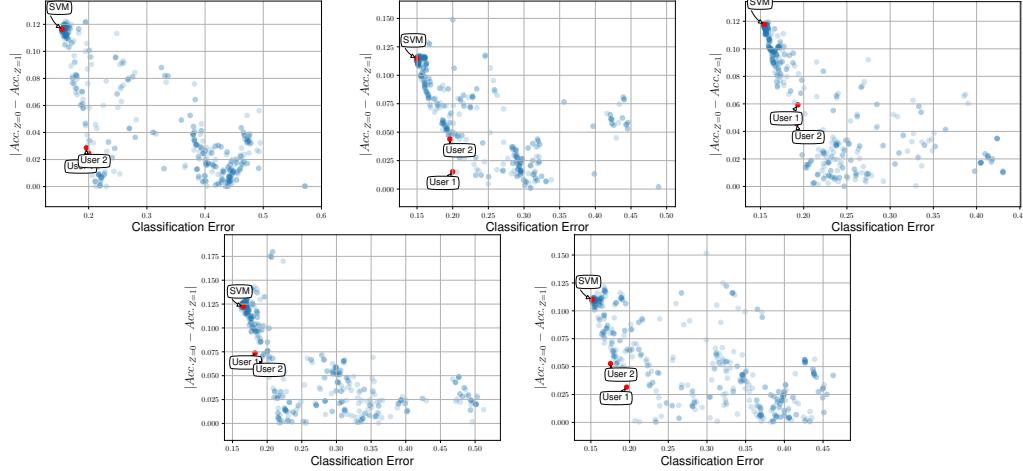


Figure 10: Adult dataset. The above operating point selection is based on 40% of the training data. In contrast to the selection in ProPublica COMPAS dataset in Figure 9, User 1 now has an inclination to be more strict in being fair and accepts a loss in accuracy in comparison to the User 2.