Multilinear Subspace Regression: An Orthogonal Tensor Decomposition Approach

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LABSP: http://www.bsp.brain.riken.jp/
Multilinear regression and applications

- **Tensor** representation of multidimensional data
  - EEG, ECoG (spatial, temporal, frequency, epoch,...)
  - Physical meaning - ease of interpretation

- From multivariate to multi-way array processes - partial least squares (PLS)

- Prediction of ECoG from scalp EEG recorded simultaneously

- Standard PLS applied on **matricization** of both X and Y
  - Small sample size problem
  - Overfitting problem (high dimension of subspace basis)
  - Lack of physical interpretation for loadings

X = Scalp EEG  Tensor regression model  Y = Intracranial ECoG
Proposed approach

**Objective function**

\[
\min_{\{P^{(n)}, Q^{(m)}\}} \left\| X - [G; t, P^{(1)}, \ldots, P^{(N-1)}] \right\|^2 + \left\| Y - [D; t, Q^{(1)}, \ldots, Q^{(M-1)}] \right\|^2 \\
\text{s. t. } \{P^{(n)T}P^{(n)}\} = I_{L_n+1}, \quad \{Q^{(m)T}Q^{(m)}\} = I_{K_m+1},
\]

**Extension of PLS to higher-order tensor data - HOPLS**

- **Goal:** to predict a tensor \( Y \) from a tensor \( X \)
- **Approach:** to extract the common latent variables

**Properties:**
- **Flexible multilinear regression framework**
- **Projection on tensor subspace basis**
- **Efficient optimization algorithm using HOOI on the \( n \)-mode cross-covariance tensor**
Key advantages

Small sample size

Robustness against overfitting and noise

HOPLS: better prediction performance and enhanced robustness to noise

Stability of the performance of HOPLS, NPLS and PLS for a varying number of latent vectors under different noise conditions

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