Supplementary material

A Practical considerations with hardware

Reference design. On the OPU system described in Figure 1, the DMD only lets us encode and randomly project binary signals. Therefore, all pairwise differences $(\boldsymbol{x}_q - \boldsymbol{x}_r)$ between the columns of $\boldsymbol{X} \in \mathbb{R}^{N \times Q}$ must be binary. To design reference signals we first collect all frames $\{\boldsymbol{\xi}_s\}_{s=1}^{S}$ and sum them $\sum_{s=1}^{S} \boldsymbol{\xi}_s$. The first reference $\boldsymbol{r}_1$ is initialized to ones at the indices where $\sum_{s=1}^{S} \boldsymbol{\xi}_s$ is nonzero. Next, some of $\boldsymbol{r}_1$'s zero-valued entries are flipped to one with probability α. A similar process is used for all subsequent references. In general, a reference $\boldsymbol{r}_q$ is initialized by assigning ones to the nonzero support of $\left(\sum_{s=1}^{S} \boldsymbol{\xi}_s + \sum_{k=1}^{q-1} \boldsymbol{r}_k\right)$ and then flipping some of its zero entries with probability α.

There is a tradeoff between large and small α. If α is too large, a reference may become all-ones before all subsequent references are generated. On the other hand if α is too small, $\boldsymbol{r}_{q+1} - \boldsymbol{r}_q$ will have many zeros and so $|\boldsymbol{A}(\boldsymbol{r}_{q+1} - \boldsymbol{r}_q)|^2$ may not be high enough to be detected by the camera sensor. The consequence of this tradeoff is that in practice the number of anchors is limited. Furthermore, in general a larger N makes it easier to make K good anchors as α can be larger which keeps $|\boldsymbol{A}(\boldsymbol{r}_{q+1} - \boldsymbol{r}_q)|^2$ away from the sensitivity threshold.

Figure 5 shows binary references reshaped into squares which were used for the linearity experiment on the OPU in Figure 2c. Here, $N = 64^2$ and $\alpha = 0.2$. The number on top of each reference is the difference in the number of ones between itself and the previously generated reference.

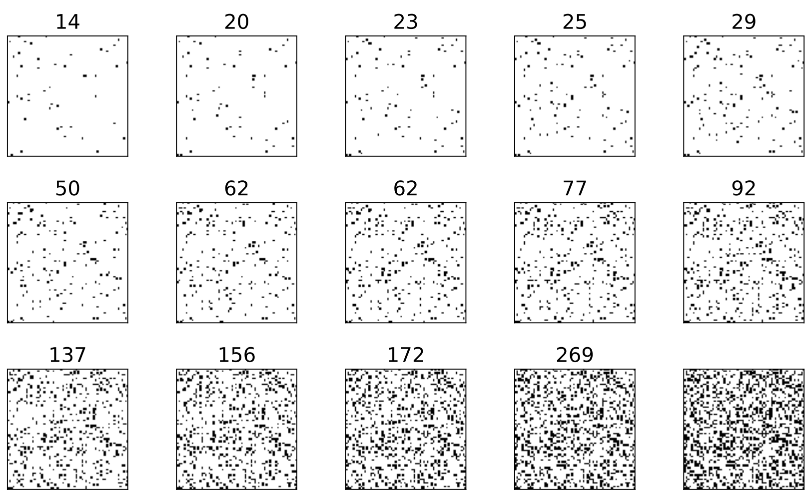

Figure 5: Binary references reshaped into squares which were used for the linearity experiment on the OPU in Figure 2c. Here $N = 64^2$ and $\alpha = 0.2$. The number on top of each anchor is the difference in the number of ones between itself and the previously generated reference.

Minimum attainable measurement. To determine the sensitivity threshold, τ, we randomly project an all-zero signal a few times and record the output. The minimum, mode or mean of these measurements can be taken to be the minimum that can be measured. Once τ is estimated, we apply a mask and zero any measurements which are equal to or less than the minimum.

Camera sensor saturation. It is possible for the signal reaching the camera to saturate the sensor. In a b-bit system we can detect this if many measurements are equal to $2^b - 1$. In all experiments we ensure that there is no saturation by adjusting the camera exposure. This again involves a tradeoff: if exposure is too high, we saturate the sensor; if it is too low, measurements may be too small to be detected and we are not exploiting the full dynamic range.

B Randomized singular value decomposition (RSVD) details

Algorithm 2 is the prototype randomized SVD algorithm given by [8]. To implement this on hardware we replace Step 1 and 2 to formulate Algorithm 3. As $\boldsymbol{A}$ in Algorithm 3 has iid entries following a standard complex Gaussian, calculating $\boldsymbol{P}$ in Algorithm 3 is the same as doing step 2 in Algorithm 2. We only need to do half the number of projections because we use an iid complex random matrix. The real and imaginary parts are two random projections.

Algorithm 2 Prototype randomized SVD algorithm [8].

Input: Matrix, $\boldsymbol{B} \in \mathbb{R}^{M \times N}$ whose SVD is required, a target number of K singular vectors
Output: The SVD $\boldsymbol{U}$, $\boldsymbol{\Sigma}$ and $\boldsymbol{V}^*$

1: Generate an $N \times 2K$ random Gaussian matrix $\boldsymbol{A}$.
2: Form $\boldsymbol{Y} = \boldsymbol{B}\boldsymbol{A}$.
3: Construct a matrix $\boldsymbol{Q}$ whose columns form an orthonormal basis for the range of $\boldsymbol{Y}$.
4: Form $\boldsymbol{C} = \boldsymbol{Q}^*\boldsymbol{B}$.
5: Compute the SVD of the smaller $\boldsymbol{C} = \tilde{\boldsymbol{U}}\boldsymbol{\Sigma}\boldsymbol{V}^*$.
6: $\boldsymbol{U} = \boldsymbol{Q}\tilde{\boldsymbol{U}}$.

Algorithm 3 Randomized SVD algorithm on the OPU.

Input: Matrix, $\boldsymbol{B} \in \mathbb{R}^{M \times N}$ whose SVD is required, a target number of K singular vectors
Output: The SVD $\boldsymbol{U}$, $\boldsymbol{\Sigma}$ and $\boldsymbol{V}^*$

1: Solve $|\boldsymbol{Y}|^2 = |\boldsymbol{A}\boldsymbol{B}^*|^2$ by treating each column of $\boldsymbol{B}^*$ as a frame and using Algorithm 1, where $\boldsymbol{A} \in \mathbb{C}^{K \times N}$ is as in the MPR problem and has K rows.
2: Horizontally stack the real and imaginary parts of $\boldsymbol{Y}^* \in \mathbb{C}^{M \times K}$ as $\boldsymbol{P} = [\mathrm{Re}(\boldsymbol{Y}^*) \quad \mathrm{Im}(\boldsymbol{Y}^*)] \in \mathbb{R}^{M \times 2K}$.
3: Construct a matrix $\boldsymbol{Q}$ whose columns form an orthonormal basis for the range of $\boldsymbol{P}$.
4: Form $\boldsymbol{C} = \boldsymbol{Q}^*\boldsymbol{B}$.
5: Compute the SVD of the smaller $\boldsymbol{C} = \tilde{\boldsymbol{U}}\boldsymbol{\Sigma}\boldsymbol{V}^*$.
6: $\boldsymbol{U} = \boldsymbol{Q}\tilde{\boldsymbol{U}}$.

C Localization with known anchor positions

If we have perfect knowledge of the anchor locations in the complex plane, we do not need to localize them for each frame s. The localization problem then boils down to multilateration, which can be formulated by minimizing the square-range-based least squares (SR-LS) objective [2],

$$\widehat{\boldsymbol{v}}_1 = \min_{\boldsymbol{v}_1} \sum_{q=2}^{Q} \left(\|\boldsymbol{v}_1 - \boldsymbol{v}_q\|_2^2 - d_q^2 \right)^2 \tag{9}$$

where $\boldsymbol{v}_1$ and $\boldsymbol{v}_q$ are as defined in Section 2.3 and d_q is the noisy measured distance. There exists efficient algorithms which solve (9) to global optimality [2], as well as suboptimal solutions based on solving a small linear system [22].