Supplemental Material: Bayesian Spike-Triggered Covariance Analysis

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Derivation of ML estimates

Here we derive the ML estimate for the STA/STC model parameters (a, b, C) based on the expected log-likelihood (eq. 6). The estimates can be differentiating $\tilde{\mathcal{L}}$ with respect to the parameters, setting them to zero, and solving:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial a} = 1 - \frac{n}{n_{sp}} |I - \Phi C|^{-\frac{1}{2}} \exp\left(\tfrac{1}{2} b^\top (\Phi^{-1} - C)^{-1} b\right) \exp(\tilde{a}_{\mathrm{ml}}) = 0 \tag{1}$$

$$\implies e^{\tilde{a}_{\mathrm{ml}}} = \left[\frac{n}{n_{sp}} |I - \Phi C|^{-\frac{1}{2}} \exp\left(\tfrac{1}{2} b^\top (\Phi^{-1} - C)^{-1} b\right)\right]^{-1} \tag{2}$$

$$\frac{\partial \tilde{\mathcal{L}}}{\partial b} = \mu - \frac{n}{n_{sp}} |I - \Phi C|^{-\frac{1}{2}} e^{\tilde{a}_{\mathrm{ml}}} \exp\left(\tfrac{1}{2} \tilde{b}_{\mathrm{ml}}^\top (\Phi^{-1} - C)^{-1} \tilde{b}_{\mathrm{ml}}\right) (\Phi^{-1} - C)^{-1} \tilde{b}_{\mathrm{ml}} \tag{3}$$

$$= \mu - (\Phi^{-1} - C)^{-1} \tilde{b}_{\mathrm{ml}} = 0$$

$$\implies \tilde{b}_{\mathrm{ml}} = \left(\Phi^{-1} - C\right) \mu \tag{4}$$

$$\begin{aligned}
\frac{\partial \tilde{\mathcal{L}}}{\partial C} &= \tfrac{1}{2}(\Lambda + \mu\mu^\top) - \frac{n}{n_{sp}} |I - \Phi C|^{-\frac{1}{2}} \exp(a) \exp\left(\tfrac{1}{2} b^\top (\Phi^{-1} - C)^{-1} b\right) \\
&\quad \cdot \left(\tfrac{1}{2} (\Phi^{-1} - C)^{-1} b (b (\Phi^{-1} - C)^{-1})^\top\right) - \frac{n}{n_{sp}} \left(-\tfrac{1}{2}\right) |I - \Phi C|^{-\frac{3}{2}} \\
&\quad \cdot |I - \Phi C| (I - \Phi C)^{-1} (-\Phi) \exp(a) \exp\left(\tfrac{1}{2} b^\top (\Phi^{-1} - C)^{-1} b\right) && (5) \\
&= \tfrac{1}{2}(\Lambda + \mu\mu^\top) - \tfrac{1}{2} (\Phi^{-1} - C)^{-1} b (b (\Phi^{-1} - C)^{-1})^\top - \tfrac{1}{2} (I - \Phi C)^{-1} \Phi = 0 && (6)
\end{aligned}$$

$$\implies \tilde{C}_{\mathrm{ml}} = \Phi^{-1} - \Lambda^{-1} \tag{7}$$

$$\tilde{b}_{\mathrm{ml}} = \Lambda^{-1} \mu \tag{8}$$

$$e^{\tilde{a}_{\mathrm{ml}}} = \tfrac{n_{sp}}{N} \left|\Phi \Lambda^{-1}\right|^{\frac{1}{2}} \exp\left(-\tfrac{1}{2} \mu^\top \Phi^{-1} \Lambda^{-1} \mu\right). \tag{9}$$