A Proof of Theorem 1

Proof. We start by noting that (17) and (18) are equivalent to

$$I_1[f^{-1}(v)] = -\phi(v) \quad (31)$$

$$I_{-1}[f^{-1}(v)] = -\phi(-v), \quad (32)$$

or

$$I_{-1}[f^{-1}(v)] = I_1[f^{-1}(-v)]. \quad (33)$$

Using (19)

$$I_{-1}[f^{-1}(v)] = I_1[1 - f^{-1}(v)] \quad (34)$$

and

$$I_{-1}[\eta] = I_1[1 - \eta]. \quad (35)$$

From (11) and (12), it follows that (17) and (18) hold if and only if

$$J(\eta) - J(1 - \eta) = \eta[J'(\eta) + J'(1 - \eta)]. \quad (36)$$

Assume that (20) holds. Taking derivatives on both sides, $J'(\eta) = -J'(1-\eta)$, (36) holds and, thus, so do (17) and (18). To show the converse, assume that (36) holds. This implies that $J(0) = J(1)$. To show that (20) holds for $\eta \notin \{0, 1\}$, we take derivatives on both sides of (36), which leads to

$$J''(\eta) = J''(1 - \eta). \quad (37)$$

This implies that

$$J'(\eta) = -J'(1 - \eta) + k \quad (38)$$

for some constant k. Since, from (36), $J'(1/2) = 0$ it follows that $k = 0$. This implies that

$$J(\eta) = J(1 - \eta) + k \quad (39)$$

for some constant k. From $J(0) = J(1)$ it follows that $k = 0$, showing that (20) holds. Finally, (21) follows from (31) and (11). □